

Math 233 - Quiz 8

October 26, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Suppose you were given a function $w = f(x, y, z)$. Name two third-order mixed partial derivatives that you would expect to be equal, and state the condition(s) under which they will be equal.

$$f_{xyz} = f_{yzx} \quad \text{IF THEY ARE BOTH CONTINUOUS} \\ \text{ON A REGION AROUND } (x, y, z)$$

2. (7 points) Use differentials to approximate the change in

$$T = \frac{2\pi\sqrt{L}}{\sqrt{g}} = 2\pi(L)^{1/2}(g)^{-1/2}$$

as (g, L) changes from $(32, 2.5)$ to $(32.03, 2.48)$.

$$dT = T_L dL + T_g dg = \pi L^{-1/2} g^{-1/2} dL + -\pi L^{1/2} g^{-3/2} dg$$

$$\Delta T \approx \frac{\pi}{\sqrt{Lg}} \Delta L - \frac{\pi\sqrt{L}}{\sqrt{g^3}} \Delta g, \quad \begin{array}{l} L = 2.5 \quad \Delta L = -0.02 \\ g = 32 \quad \Delta g = 0.03 \end{array}$$

$$\Delta T \approx \frac{\pi}{\sqrt{(32)(2.5)}} (-0.02) - \frac{\pi\sqrt{2.5}}{\sqrt{(32)^3}} (0.03)$$

$$\approx -0.00785$$