Math 233 - Test 1

Name _____

September 14, 2023

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) In this problem, the force vectors $\vec{F_1}$ and $\vec{F_2}$ are 2D vectors in the xy-plane.

   (a) The force $\vec{F_1}$ has magnitude 50 and makes a 120° angle with the positive x-axis. Find the component form of $\vec{F_1}$.

   (b) The force $\vec{F_2}$ has component form $\vec{F_2} = 30\hat{\imath} - 30\sqrt{3}\,\hat{\jmath}$. What angle does $\vec{F_2}$ make with the positive x-axis?

   (c) Refer to parts (a) and (b). Compute the resultant vector $\vec{F} = \vec{F_1} + \vec{F_2}$.

   (d) Refer to part (c). What angle does $\vec{F}$ make with the positive x-axis?

2. (4 points) Explain how the right-hand rule gives the orientation of the coordinate axes in a 3-dimensional rectangular coordinate system.

3. (6 points) Suppose that θ is the angle between the two nonzero vectors $\vec{u}$ and $\vec{w}$. What can you say about $\vec{u} \cdot \vec{w}$ in each of these cases?

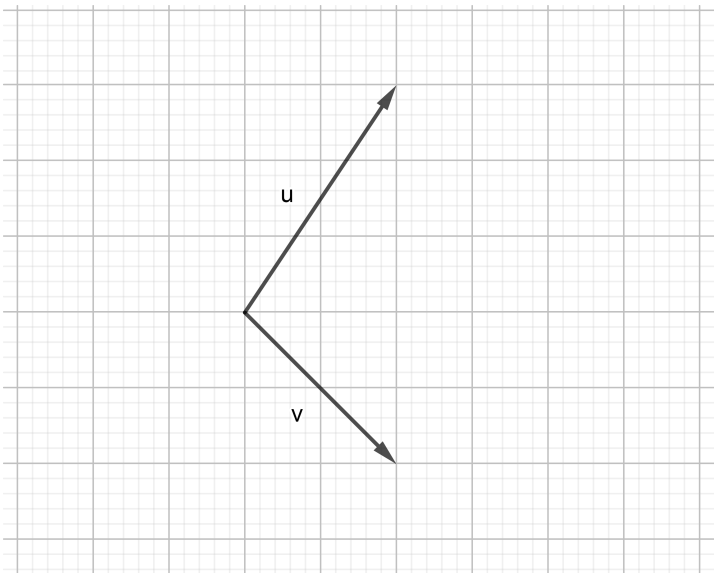
   (a) θ is a right angle.

   (b) θ is an acute angle.

   (c) θ is an obtuse angle.

4. (6 points) Find a vector of magnitude 6 that has the direction from $P\ (-2, 4, -3)$ to $Q\ (-5, 3, 4)$.

5. (6 points) The figure below shows the vectors $\vec{u}$ and $\vec{v}$. Sketch and label the vectors $\vec{u} + \frac{1}{2}\vec{v}$ and $\vec{v} - \vec{u}$, and illustrate how your vectors follow from the parallelogram law.



6. (6 points) Let $\vec{x} = -9\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$.

   (a) Find a vector, different from $\vec{x}$, that is parallel to $\vec{x}$. Give a one-sentence explanation for how you know.

   (b) Find a nonzero vector that is orthogonal to $\vec{x}$. Give a one-sentence explanation for how you know.

7. (10 points) For this problem, you will need to use that the distance from a point Q to the line passing through P and parallel to $\vec{v}$ is given by

$$D = \frac{\|\vec{PQ} \times \vec{v}\|}{\|\vec{v}\|}.$$

   (a) First choose any point on the line described by the parametric equations below. Let your point be Q. (There are infinitely many choices for Q.)

$$x = 3t - 4, \quad y = -5t, \quad z = t + 5.$$

   (b) Now consider the line ℓ with symmetric equations

$$\frac{x+6}{2} = y - 3 = \frac{z-1}{-3}.$$

   Find a point P on ℓ and a vector $\vec{v}$ parallel to ℓ.

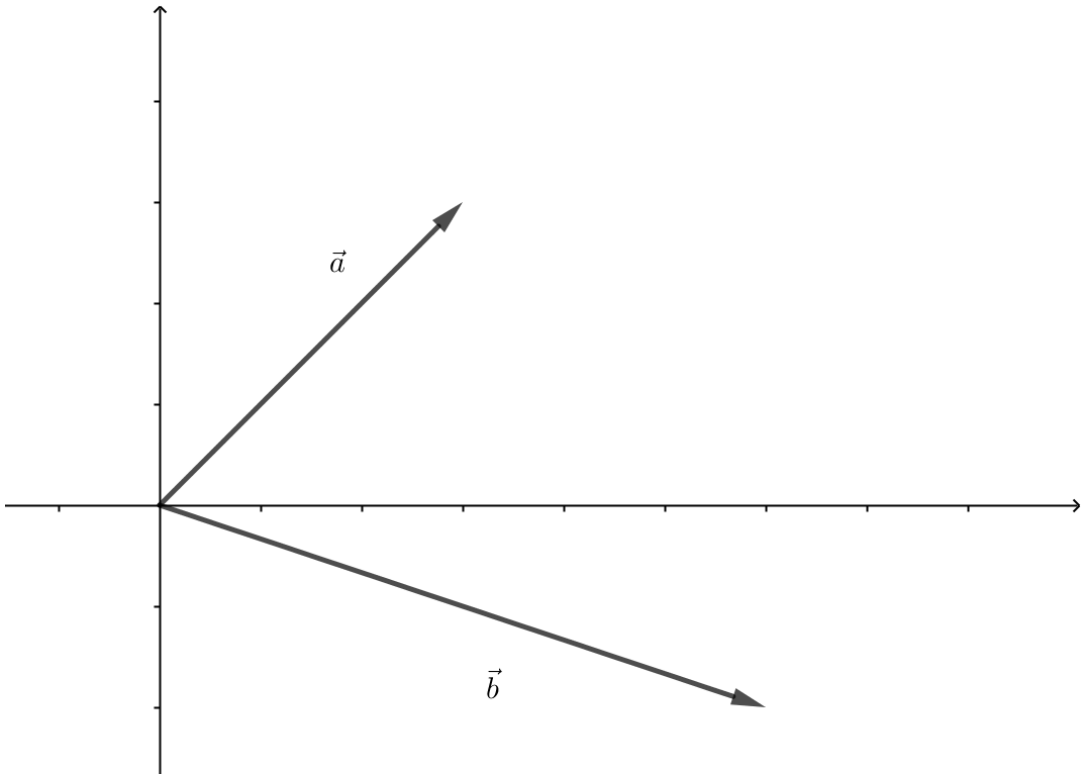
   (c) Compute the distance from Q to the line ℓ.

8. (8 points) Find the angle between the planes described by the equations below. Write your final answer in degrees rounded to the nearest hundredth.

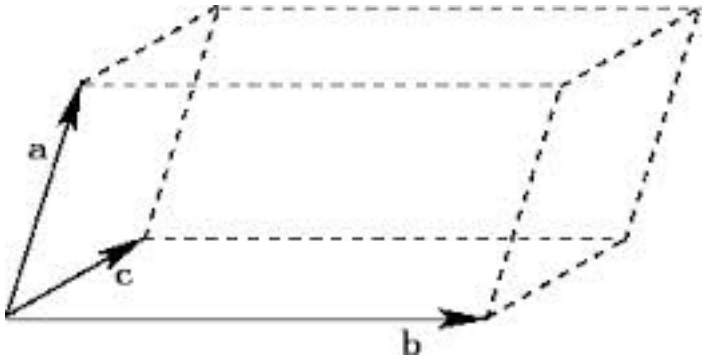
$$2x - y + 2z = 7 \qquad\qquad -5x + 3z = 12$$

9. (4 points) Find the projection of $\vec{w} = \hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ onto $\vec{u} = 7\hat{\imath} + 4\hat{k}$.

10. (4 points) The figure below shows the vectors $\vec{a}$ and $\vec{b}$. Sketch $\text{proj}_{\vec{b}}\,\vec{a}$.



11. (8 points) A crystal structure has the form of a parallelepiped determined by the vectors $\vec{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\vec{b} = 3\hat{\jmath} + 5\hat{k}$, and $\vec{c} = -4\hat{\imath} + 2\hat{\jmath} + \hat{k}$, where distances are measured in micrometers. Find the volume of the parallelepiped.



12. (6 points) Let $\vec{r}(t) = \dfrac{\sin t}{t}\hat{\imath} + \ln(t+1)\hat{\jmath} + e^{2t}\hat{k}$.

    (a) Determine the domain of $\vec{r}$.

    (b) Compute $\lim\limits_{t\to 0} \vec{r}(t)$.

13. (4 points) Explain how to find a vector that is orthogonal to each vector in a pair of non-parallel vectors.

14. (8 points) Let $\vec{r}(t) = 3\sin t\,\hat{\imath} - 3\cos t\,\hat{\jmath} + 4\,\hat{k}$.

(a) Compute $\|\vec{r}(t)\|$.

(b) Determine the derivative $\vec{r}\,'(t)$.

(c) Compute $\vec{r}(t) \cdot \vec{r}\,'(t)$.

(d) Compute $\vec{r}(t) \times \vec{r}\,'(t)$.

15. (10 points) Find an equation of the plane passing through the points $R\,(1, -2, 4)$, $S\,(0, 3, -5)$, and $T\,(8, 2, -3)$.