

Math 233 - Test 2
October 12, 2023

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (4 points) Describe, in detail, the graph of the vector-valued function

$$\vec{r}(t) = (1 + t)\hat{i} + (2 + 5t)\hat{j} + (-1 + 6t)\hat{k}.$$

2. (8 points) Consider the vector-valued function $\vec{r}(t) = (t + 2)\hat{i} + (t^2 - 3)\hat{j}$.

- (a) Write a corresponding set of parametric equations whose graph is the same as that of \vec{r} .

- (b) Eliminate the parameter t from your parametric equations above to find an equation for the graph in terms of the variables x and y .

- (c) Sketch the graph of \vec{r} and use arrows to indicate the orientation.

3. (10 points) An object is launched from the point $P(0, 1, -1)$ with initial velocity vector $\vec{v}(0) = \hat{i} - 2\hat{j} + \hat{k}$. The object undergoes a constant acceleration of $\vec{a}(t) = \hat{j} + 2\hat{k}$. Find the object's location at $t = 4$.

4. (10 points) Set up the definite integral that gives the length of the graph of

$$\vec{r}(t) = (3t^2 + 1)\hat{i} + (4t^2 - 1)\hat{j} + 4t^3\hat{k}$$

from $t = 0$ to $t = 2$. Evaluate your integral by hand. (If you've done everything correctly, your integral should require a simple u -substitution.)

5. (10 points) Let $\vec{r}(t) = (t - 3)\hat{i} + (2t - 4)\hat{j} + 2t\hat{k}$. Reparameterize \vec{r} in terms of the arc-length parameter starting from $t = 3$.

6. (8 points) Show that the curvature at any point on the graph of $\vec{r}(t) = -2\sin(2t)\hat{i} - 2\cos(2t)\hat{j}$ is $1/2$.

7. (8 points) Sketch the graph of the vector-valued function $\vec{r}(t) = -2\sin(2t)\hat{i} - 2\cos(2t)\hat{j}$. Then, on your graph, sketch the vectors $\hat{T}(0)$ and $\hat{N}(0)$.

8. (15 points) A projectile is fired into the air from ground level with an initial speed of 500 m/sec at an angle of 60° with the horizontal. (Use $g = 9.8$ m/sec.)

(a) Find the maximum height of the projectile.

(b) What is the range of the projectile?

(c) Show that the speed of the projectile when it hits the ground is 500 m/sec.

9. (12 points) Each of these equations defines a surface in 3-space. Describe each surface.

(a) $4y = x^2 + 8z^2$

(b) $z = 9x - 7y + 13$

(c) $x^2 + y^2 = 4$

(d) $2x^2 + 8y^2 + z^2 = 16$

10. (10 points) Let $G(x, y, z) = \sqrt{x^2 + y^2 - z}$.

(a) Evaluate $G(-4, 5, -8)$.

(b) What is the domain of G ?

(c) Describe or sketch the level surface $G(x, y, z) = 0$.

(d) Describe or sketch the level surface $G(x, y, z) = -1$.

(e) Describe or sketch the level surface $G(x, y, z) = 1$.

11. (5 points) Find the curvature function for $y = e^x$. What happens to the curvature as $x \rightarrow \infty$? Explain how/why your answer is obvious from the graph of $y = e^x$.