Math 233 - Test 3

November 9, 2023

Name ____________________

Score __________

Show all work to receive full credit. Supply explanations where necessary.

1. (14 points) Determine the limit or show that it does not exist.

   (a) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{y^2 - xy}{\sqrt{x} - \sqrt{y}}$

   (b) $\displaystyle\lim_{(x,y)\to(0,2)} \frac{x^2(y-2)}{x^4 + (y-2)^2}$

2. (2 points) For the function F, under what conditions would you expect $F_{xyz}(x, y, z) = F_{yzx}(x, y, z)$?

3. (7 points) Show that f is not continuous at $(0, 0)$.

$$f(x,y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 1, & (x,y) = (0,0) \end{cases}$$

4. (8 points) Let $u = e^{-2t} \sin x \, \sin y$. Show that $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}$.

5. (3 points) The graph of $g(x,y) = \sqrt{1 - x^2 - y^2}$ is the upper half of the unit sphere centered at the origin. Think about the graph at the point $(1/2, 0)$. Without actually computing it, tell me the sign of $g_x(1/2, 0)$ and say how you know.

6. (8 points) Find the linearization of $f(x, y) = \sqrt{41 - 4x^2 - y^2}$ at $(x, y) = (2, 3)$. Then use your linearization to approximate $f(2.1, 2.9)$.

7. (5 points) Suppose w is a function of x, y, z and x, y, z are functions of s, t. Write the chain rule formulas for $\dfrac{\partial w}{\partial s}$ and $\dfrac{\partial w}{\partial t}$.

8. (5 points) Assume that y is implicitly defined as a function of x by the equation $xe^y + ye^x = 2x^2y$. Use partial derivatives to find dy/dx.

9. (6 points) The electric voltage in a certain region in space is described by the function $V(x, y, z) = 5x^2 - 3xy + xyz$. At the point $(3, 4, 5)$, in what direction is the voltage increasing most rapidly? What is the rate of change in that direction?

10. (8 points) Identify each quadric surface.

    (a) $5x^2 - 4y^2 + 20z^2 = 0$

    (b) $-x^2 + 36y^2 + 36z^2 = 9$

    (c) $z - 4x^2 + y^2 = 0$

    (d) $-3x^2 + 5y^2 - z^2 = 10$

11. (6 points) Find an equation of the plane tangent to the surface $x^3 + y^3 = 3xyz$ at the point $(1, 2, 3/2)$.

12. (8 points) Find the critical points of the function $f(x, y) = x^3 - 3xy + y^2 + y - 5$.

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The following problems are due Tuesday, November 14, 2023. You must work on your own.

13. (6 points) Newton's Law of Universal Gravitation describes the gravitational force of attraction between two bodies with masses M and m:

$$F = G\frac{Mm}{r^2},$$

where r is the distance between the bodies and G is a universal constant. For the Sun and Earth, we have the following:

$$M = 2 \times 10^{30}\,\mathrm{kg}, \quad \Delta M = 7 \times 10^{25}\,\mathrm{kg}$$

$$m = 6 \times 10^{24}\,\mathrm{kg}, \quad \Delta m = 6 \times 10^{20}\,\mathrm{kg}$$

$$r = 1.5 \times 10^{11}\,\mathrm{m}, \quad \Delta r = 6 \times 10^{8}\,\mathrm{m}$$

With $G = 6.7 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$, use differentials to approximate ΔF.

14. (6 points) Use the **definition of differentiability** to show that $g(x, y) = 5xy - x^2$ is differentiable everywhere in $\mathbb{R}^2$.

15. (8 points) Find and classify the critical points of $g(x, y) = x^2y - x^2 - 8y^2 + 10$.