**Math 233 - Final Exam A** December 8, 2023

Name key

Score

**Show all work to receive full credit.** Supply explanations where necessary. This portion of the test is due December 14. You must work individually.

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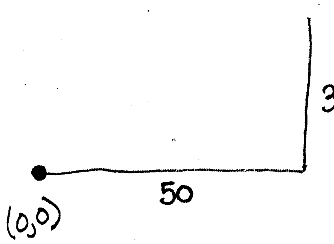
1. (10 points) A projectile is launched from the origin, which is a point 50 ft to the left of a 30-ft vertical cliff. It is launched toward the cliff at a speed of $50\sqrt{2}$ ft/sec at an angle of 45° to the horizontal. Assume the ground is horizontal on the top of the cliff and the only force acting on the projectile is gravity ($g = 32\,\text{ft/sec}^2$).

   (a) Find the coordinates of the landing spot of the projectile on the top of the cliff.




$$\vec{r}(t) = 50\sqrt{2}\cos 45^\circ\, t\,\hat{i} + \left(-16t^2 + 50\sqrt{2}\sin 45^\circ\, t\right)\hat{j}$$

$$= 50t\,\hat{i} + \left(-16t^2 + 50t\right)\hat{j}$$

$$-16t^2 + 50t = 30$$

$$8t^2 - 25t + 15 = 0$$

$$t = \frac{25 \pm \sqrt{625-480}}{16} = \frac{25 \pm \sqrt{145}}{16}$$

$$t = \frac{25+\sqrt{145}}{16} \approx 2.3151 \text{ sec}$$

COORDS ARE ABOUT

$(115.755, 30)$

   (b) What is the maximum height of the projectile?

$$-32t + 50 = 0$$

$$\Downarrow$$

$$t = \frac{25}{16}$$

$$-16\left(\frac{25}{16}\right)^2 + 50\left(\frac{25}{16}\right) = \frac{25^2}{16} = 39.0625 \text{ FT}$$

   (c) What is the time of flight?

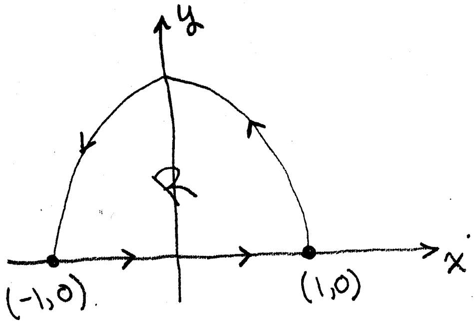
DETERMINED IN PART (a):

$$t = \frac{25+\sqrt{145}}{16} \text{ SEC}$$

$$\approx 2.3151 \text{ SEC}$$

2. (10 points) Let $C$ be the closed curve that follows the graph of $y = 1 - x^2$ from $(1, 0)$ to $(-1, 0)$ and then follows the $x$-axis from $(-1, 0)$ to $(1, 0)$.

(a) Use Green's theorem to evaluate $\int_C y^2\,dx + xy\,dy$. $= \iint_R (y - 2y)\,dA$




$$= \int_{-1}^{1}\int_{0}^{1-x^2} -y\,dy\,dx = -\frac{1}{2}\int_{-1}^{1}(1-x^2)^2\,dx$$

$$= -\frac{1}{2}\left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right)\Big|_{-1}^{1}$$

$$= -\frac{1}{2}\left[\left(1 - \frac{2}{3} + \frac{1}{5}\right) - \left(-1 + \frac{2}{3} - \frac{1}{5}\right)\right]$$

$$= -\frac{1}{2}\left(\frac{16}{15}\right) = -\frac{8}{15}$$

(b) Evaluate $\int_C y^2\,dx + xy\,dy$ by parameterizing $C$.

$C:\ x = -t,\quad y = 1 - t^2 \qquad -1 \le t \le 1$

AND THEN

$x = t,\quad y = 0 \qquad -1 \le t \le 1$

$$\int_{-1}^{1}(1-t^2)^2(-1)\,dt + (-t)(1-t^2)(-2t\,dt) + \int_{-1}^{1} 0\,dt + 0$$

$$= \int_{-1}^{1}(-3t^4 + 4t^2 - 1)\,dt = -\frac{3}{5}t^5 + \frac{4}{3}t^3 - t\,\Big|_{-1}^{1}$$

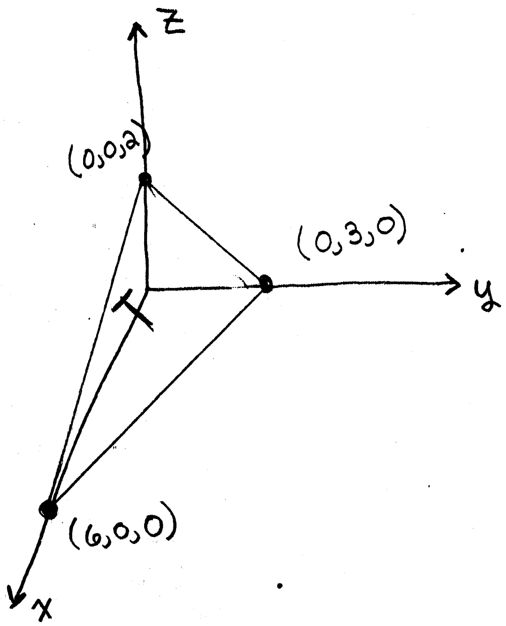
$$= \left(-\frac{3}{5} + \frac{4}{3} - 1\right) - \left(\frac{3}{5} - \frac{4}{3} + 1\right)$$

$$= \left(-\frac{4}{15}\right) - \left(\frac{4}{15}\right) = -\frac{8}{15}$$

(c) Look at your result from part (a). Is $\vec{F}(x, y) = y^2\,\hat{\imath} + xy\,\hat{\jmath}$ a conservative vector field? Briefly explain.

NO WAY. THE CURVE $C$ STARTS AND ENDS AT $(1,0)$.
IF $\vec{F}$ WAS CONSERVATIVE, THE LINE INTEGRAL WOULD BE ZERO

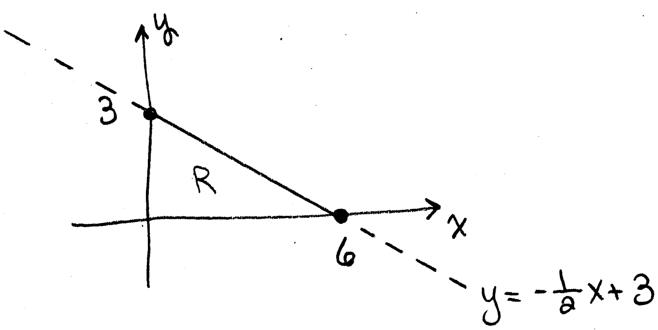
3. (10 points) Let $T$ be the tetrahedron in space bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + 2y + 3z = 6$. Set up the triple integrals required to compute the average value of $f(x, y, z) = x + y + z$ on $T$. Use a computer algebra system to evaluate the integrals and state the average value.




Volume of $T$ =

$$\underbrace{\int_{x=0}^{x=6}\int_{y=0}^{y=-\frac{1}{2}x+3}}_{R}\int_{z=0}^{z=\frac{1}{3}(6-x-2y)} dz\, dy\, dx = 6$$

$$\underbrace{\int_{x=0}^{x=6}\int_{y=0}^{y=-\frac{1}{2}x+3}}_{R}\int_{z=0}^{z=\frac{1}{3}(6-x-2y)} (x+y+z)\, dz\, dy\, dx = \frac{33}{2}$$




Avg value $= \frac{1}{6}\left(\frac{33}{2}\right) = \boxed{\frac{33}{12} = 2.75}$