

Math 233 - Final Exam A

December 8, 2023

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. This portion of the test is due December 14. You must work individually.

1. (10 points) A projectile is launched from the origin, which is a point 50 ft to the left of a 30-ft vertical cliff. It is launched toward the cliff at a speed of $50\sqrt{2}$ ft/sec at an angle of 45° to the horizontal. Assume the ground is horizontal on the top of the cliff and the only force acting on the projectile is gravity ($g = 32$ ft/sec²).

(a) Find the coordinates of the landing spot of the projectile on the top of the cliff.

(b) What is the maximum height of the projectile?

(c) What is the time of flight?

2. (10 points) Let C be the closed curve that follows the graph of $y = 1 - x^2$ from $(1, 0)$ to $(-1, 0)$ and then follows the x -axis from $(-1, 0)$ to $(1, 0)$.

(a) Use Green's theorem to evaluate $\int_C y^2 dx + xy dy$.

(b) Evaluate $\int_C y^2 dx + xy dy$ by parameterizing C .

(c) Look at your result from part (a). Is $\vec{F}(x, y) = y^2 \hat{i} + xy \hat{j}$ a conservative vector field? Briefly explain.

3. (10 points) Let T be the tetrahedron in space bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + 2y + 3z = 6$. Set up the triple integrals required to compute the average value of $f(x, y, z) = x + y + z$ on T . Use a computer algebra system to evaluate the integrals and state the average value.