Math 233 - Quiz 12

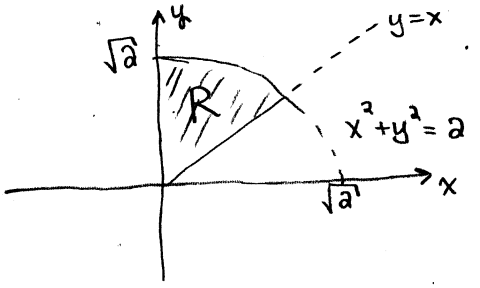
December 4, 2025

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. Use other paper as necessary. This quiz is due December 9.

1. (2 points) The iterated integral shown below is in rectangular coordinates. Convert to polar coordinates and evaluate.



$$\int_0^1 \int_x^{\sqrt{2-x^2}} (x+2y)\,dy\,dx = \int_{\theta=\pi/4}^{\theta=\pi/2} \int_{r=0}^{r=\sqrt{2}} (\cos\theta + 2\sin\theta)\, r^2\, dr\, d\theta$$

$$= \int_{\pi/4}^{\pi/2} \frac{\sqrt{8}}{3}(\cos\theta + 2\sin\theta)\,d\theta = \frac{\sqrt{8}}{3}\left(\sin\theta - 2\cos\theta\right)\Big|_{\pi/4}^{\pi/2} = \frac{\sqrt{8}}{3}\left(1 - \frac{\sqrt{2}}{2} + \sqrt{2}\right)$$

$$= \frac{\sqrt{8}}{3}\left(1 + \frac{\sqrt{2}}{2}\right)$$

2. (2 points) Find the area of the region cut from the 1st quadrant by the polar curve $r = 2\sqrt{2 - \sin 2\theta}$.

See attached graph of region.

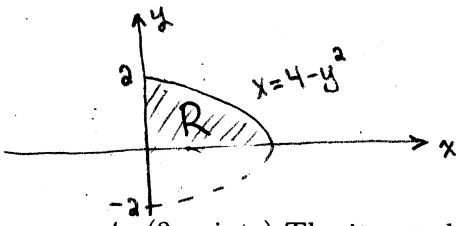
$$\text{Area} = \int_{\theta=0}^{\theta=\pi/2} \int_{r=0}^{r=2\sqrt{2-\sin 2\theta}} r\,dr\,d\theta$$

$$= 2\int_0^{\pi/2} (2 - \sin 2\theta)\,d\theta = 4\theta + \cos 2\theta \Big|_0^{\pi/2}$$

$$= (2\pi - 1) - (0 + 1)$$

$$= 2\pi - 2$$

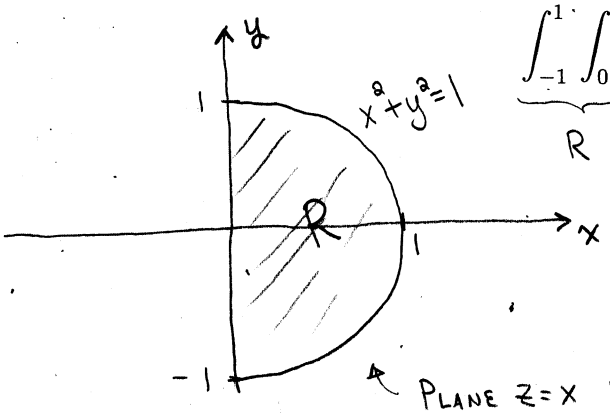
3. (3 points) A solid lies in the 1st octant inside the cylinder $x = 4 - y^2$ and below the plane $y + z = 2$. The density of the solid at the point (x, y, z) is given by $\rho(x, y, z) = 1 + x + y$. Set up the iterated integral that gives the mass of the solid. Use a computer algebra system to evaluate your integral.



$$\iiint_R \int_{z=0}^{z=2-y} (1+x+y)\,dz\,dA = \int_{y=0}^{y=2} \int_{x=0}^{x=4-y^2} \int_{z=0}^{z=2-y} (1+x+y)\,dz\,dx\,dy$$

$$= \frac{332}{15} = 22.\overline{13}$$

4. (3 points) The iterated integral shown below is in rectangular coordinates. Convert to cylindrical coordinates and evaluate.



$$\underbrace{\int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}}}_{R} \int_0^x (x^2+y^2)\,dz\,dx\,dy$$

$$= \int_{\theta=-\pi/2}^{\theta=\pi/2} \int_{r=0}^{r=1} \int_{z=0}^{z=r\cos\theta} r^3\,dz\,dr\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \int_0^1 r^4 \cos\theta\,dr\,d\theta = \frac{1}{5}\int_{-\pi/2}^{\pi/2} \cos\theta\,d\theta$$

$$= \frac{2}{5}\sin\theta\Big|_0^{\pi/2} = \frac{2}{5}$$

Plane $z = x$ lies above and crosses where $x = 0$