Math 233 - Quiz 2

August 28, 2025

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary.

1. (1 point) What does it mean for two vectors $\vec{x}$ and $\vec{y}$ to be parallel?

ONE IS A NONZERO SCALAR MULTIPLE OF THE OTHER...

$\vec{X} \parallel \vec{y} \Leftrightarrow \vec{X} = k\vec{y}$ FOR SOME SCALAR $k \neq 0$

2. (2 points) Find a vector of magnitude 15 that has the opposite direction of $\vec{PQ}$, where $P(3, 2, -5)$ and $Q(1, 4, -2)$.

$\vec{PQ} = \langle 1-3, 4-2, -2-(-5) \rangle$

$= \langle -2, 2, 3 \rangle$

$\|\vec{PQ}\| = \sqrt{4+4+9} = \sqrt{17}$

$\boxed{\dfrac{-15\vec{PQ}}{\|\vec{PQ}\|} = \dfrac{1}{\sqrt{17}}\left(30\hat{\imath} - 30\hat{\jmath} - 45\hat{k}\right)}$

3. (3 points) Determine the measure of the angle between the vectors $\vec{a} = 3\hat{\imath} + 4\hat{\jmath} - 9\hat{k}$ and $\vec{b} = 2\hat{\jmath} + 8\hat{k}$. Write your final answer in degrees, rounded to the nearest hundredth.

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\,\|\vec{b}\|} = \dfrac{0+8-72}{\sqrt{106}\,\sqrt{68}} = \dfrac{-64}{\sqrt{106}\,\sqrt{68}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{-64}{\sqrt{106}\,\sqrt{68}}\right)$

$\|\vec{a}\| = \sqrt{9+16+81}$

$\|\vec{b}\| = \sqrt{0+4+64}$

$\approx \boxed{138.92^\circ}$

4. (2 points) If $\vec{u}\cdot\vec{v} = \vec{u}\cdot\vec{w}$, must it be true that $\vec{v} = \vec{w}$? Explain.

$\vec{u}\cdot\vec{v} - \vec{u}\cdot\vec{w} = 0$

$\vec{u}\cdot(\vec{v} - \vec{w}) = 0$

AS LONG AS $\vec{v} - \vec{w}$ IS ORTHOGONAL TO $\vec{u}$, WE WILL HAVE $\vec{u}\cdot\vec{v} = \vec{u}\cdot\vec{w}$. THERE ARE INF. MANY $\vec{v}, \vec{w}$ FOR WHICH THIS IS TRUE. FOR EXAMPLE,

$\vec{u} = \langle 1, 1, 1 \rangle$, $\vec{v} = \langle 3, 5, 9 \rangle$, $\vec{w} = \langle 4, 5, 8 \rangle$.

$\vec{u}\cdot\vec{v} = 17$

$\vec{u}\cdot\vec{w} = 17$

$\vec{v} \neq \vec{w}$

5. (2 points) Find a unit vector that is orthogonal to $\vec{w} = \langle 3, -2, -1 \rangle$.

LET $\vec{u} = \langle 2, 3, 0 \rangle$

THEN $\vec{u}\cdot\vec{w} = 3(2) + (-2)(3) + (0)(-1)$

$= 0.$

SO $\vec{u}$ & $\vec{w}$ ARE ORTHOG.

NOW NORMALIZE...

$\|\vec{u}\| = \sqrt{4+9+0} = \sqrt{13}$

$\boxed{\dfrac{\vec{u}}{\|\vec{u}\|} = \dfrac{2}{\sqrt{13}}\hat{\imath} + \dfrac{3}{\sqrt{13}}\hat{\jmath}}$