Math 233 - Quiz 3

September 4, 2025

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find a unit vector that is orthogonal to both $\vec{v} = 5\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{w} = -2\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$.

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & -1 & 3 \\ -2 & -4 & 2 \end{vmatrix}$$

$$= \hat{\imath}(-2+12) - \hat{\jmath}(10+6) + \hat{k}(-20-2) = 10\hat{\imath} - 16\hat{\jmath} - 22\hat{k}$$

$$\|\vec{v} \times \vec{w}\| = \sqrt{100 + 256 + 484} = \sqrt{840} = 2\sqrt{210}$$

$$\boxed{\frac{\vec{v} \times \vec{w}}{\|\vec{v} \times \vec{w}\|} = \frac{1}{\sqrt{210}}\left(5\hat{\imath} - 8\hat{\jmath} - 11\hat{k}\right)}$$

2. (2 points) Find the projection of $\vec{v} = 3\hat{\imath} - 4\hat{\jmath} - 3\hat{k}$ onto $\vec{w} = \hat{\imath} + 6\hat{\jmath} + 2\hat{k}$.

$$\text{proj}_{\vec{w}}\, \vec{v} = \frac{\vec{v} \cdot \vec{w}}{\vec{w} \cdot \vec{w}}\, \vec{w} = \frac{3 - 24 - 6}{1 + 36 + 4}\, \vec{w} = \boxed{\frac{-27}{41}\left(\hat{\imath} + 6\hat{\jmath} + 2\hat{k}\right)}$$

3. (1 point) For vectors $\vec{x}$ and $\vec{y}$, explain why you should not expect $\text{proj}_{\vec{y}}\vec{x}$ and $\text{proj}_{\vec{x}}\vec{y}$ to be equal.

THEY HAVE COMPLETELY DIFFERENT DIRECTIONS.

$\text{proj}_{\vec{y}}\, \vec{x}$ HAS DIRECTION OF $\vec{y}$.

$\text{proj}_{\vec{x}}\, \vec{y}$ HAS DIRECTION OF $\vec{x}$.

4. (2 points) Find a set of parametric equations for the line passing through the two points $A(4, -9, 2)$ and $B(7, 3, 1)$.

$\vec{AB} = \langle 3, 12, -1 \rangle$

Using $\vec{AB}$ AND POINT A

$$\boxed{\begin{aligned} x &= 3t + 4 \\ y &= 12t - 9 \\ z &= -t + 2 \end{aligned}}$$

5. (2 points) A line is described by the equations $\dfrac{2x-4}{5} = \dfrac{3-y}{4} = z + 6$. Determine a point on the line and a vector parallel to the line.

$$\frac{x-2}{5/2} = \frac{y-3}{-4} = \frac{z-(-6)}{1}$$

$$\boxed{\begin{aligned} &\text{POINT } (2, 3, -6) \\ &\vec{v} = \tfrac{5}{2}\hat{\imath} - 4\hat{\jmath} + \hat{k} \end{aligned}}$$