

Math 233 - Quiz 4

September 18, 2025

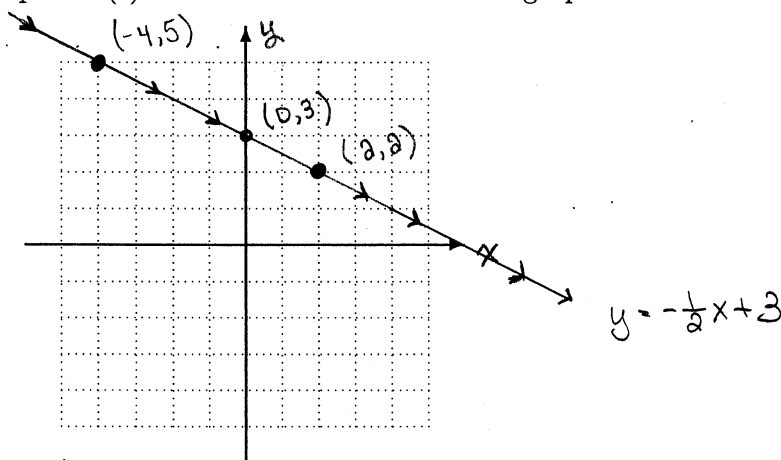
Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Let $\vec{r}(t) = (6t - 4)\hat{i} + (-3t + 5)\hat{j}$.(a) Write the set of parametric equations whose graph is that of $\vec{r}(t)$. Then eliminate the parameter t to obtain an equation in the rectangular coordinates x and y .

$$\begin{aligned} \boxed{\begin{array}{l} x = 6t - 4 \\ y = -3t + 5 \end{array}} &\rightarrow t = \frac{x+4}{6} = \frac{x}{6} + \frac{2}{3} \\ &y = -3\left(\frac{x}{6} + \frac{2}{3}\right) + 5 \Rightarrow \boxed{y = -\frac{1}{2}x + 3} \end{aligned}$$

(b) Carefully sketch the graph of $\vec{r}(t)$ and draw arrows to show the graph's orientation.2. (2 points) Let $\vec{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$. Compute the principal unit tangent vector $\hat{T}(t)$.

$$\begin{aligned} \vec{r}'(t) &= \hat{i} + 2t\hat{j} + 3t^2\hat{k} \\ \|\vec{r}'(t)\| &= \sqrt{1 + 4t^2 + 9t^4} \\ \hat{T}(t) &= \frac{\hat{i} + 2t\hat{j} + 3t^2\hat{k}}{\sqrt{1 + 4t^2 + 9t^4}} \end{aligned}$$

3. (3 points) Let $\vec{r}(t) = -2\sin 7t\hat{i} + 9\hat{j} + 2\cos 7t\hat{k}$. Compute $\|\vec{r}(t)\|$, and then show that $\vec{r}(t)$ is orthogonal to its derivative.

$$\|\vec{r}(t)\| = \sqrt{4\sin^2 7t + 81 + 4\cos^2 7t} = \sqrt{4 + 81} = \boxed{\sqrt{85}}$$

$$\vec{r}'(t) = -14\cos 7t\hat{i} - 14\sin 7t\hat{k}$$

$$\vec{r}(t) \cdot \vec{r}'(t) = 28\sin 7t \cos 7t + 0 - 28\sin 7t \cos 7t = 0$$