Math 233 - Quiz 5

September 25, 2025

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Set up the definite integral that gives the length of the graph of $\vec{r}(t) = t\hat{\imath} + t^3\hat{\jmath} + e^{2t}\hat{k}$ from $t = 0$ to $t = 1$. Use your calculator to approximate the value of your integral.

$\vec{r}\,'(t) = \hat{\imath} + 3t^2\hat{\jmath} + 2e^{2t}\hat{k}$

$$\int_0^1 \|\vec{r}\,'(t)\|\,dt = \int_0^1 \sqrt{1 + 9t^4 + 4e^{4t}}\,dt \approx 6.58$$

2. (4 points) Let $\vec{r}(t) = (2t+1)\hat{\imath} + (-2t+3)\hat{\jmath} + t\hat{k}$. Find the arc length parameter starting at $t = 1$, and then reparameterize $\vec{r}$ in terms of arc length.

$\vec{r}\,'(t) = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$

$$s(t) = \int_1^t \sqrt{4+4+1}\,d\tau = \int_1^t 3\,d\tau = 3\tau\Big|_1^t = 3t - 3$$

$s = 3t - 3 \Rightarrow t = \frac{s+3}{3}$

$$\vec{R}(s) = \left[2\left(\frac{s+3}{3}\right) + 1\right]\hat{\imath} + \left[-2\left(\frac{s+3}{3}\right) + 3\right]\hat{\jmath} + \frac{s+3}{3}\hat{k}$$

3. (1 point) Referring to problem 2, determine the curvature function for $\vec{r}(t)$.

$$\hat{T}(t) = \frac{2\hat{\imath} - 2\hat{\jmath} + \hat{k}}{3} = \text{A CONSTANT VECTOR}$$

$$\kappa(t) = \frac{\|\hat{T}'(t)\|}{\|\vec{r}\,'(t)\|} = \frac{0}{3} = 0 \qquad \kappa(t) = 0$$

4. (1 point) What is the curvature at any point on a circle of radius 5?

$\kappa = \frac{1}{5}$

THE CURVATURE ANYWHERE ON A CIRCLE OF RADIUS r IS $\frac{1}{r}$.