

Math 233 - Quiz 6

October 2, 2025

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) What are the three main properties of the principal unit normal vector.

① $\hat{N}(t)$ IS A UNIT VECTOR

② $\hat{N}(t)$ IS NORMAL TO THE CURVE (ORTHOG. TO $\hat{T}(t)$)

③ $\hat{N}(t)$ POINTS IN THE DIRECTION $\hat{T}(t)$ IS CHANGING (POINTS "INTO" THE CURVE)

2. (1 point) Refer to one of your properties from above to explain why $\hat{N}(t)$ does not exist for a line.

LOOK AT ③...

FOR MOTION ALONG A LINE, $\hat{T}(t)$ IS NOT CHANGING.

3. (6 points) A projectile is fired from ground level at an angle of 12° with the horizontal. The projectile is to have a range of 200 feet. Find the required initial speed and the maximum height of the projectile. (Use $g = 32 \text{ ft/s}^2$.)

$$\vec{r}(t) = v_0 \cos 12^\circ t \hat{i} + (-16t^2 + v_0 \sin 12^\circ t) \hat{j}$$

$$v_0 \cos 12^\circ t = 200 \Rightarrow v_0 = \frac{200}{t \cos 12^\circ}$$

$$-16t^2 + v_0 \sin 12^\circ t = 0$$

$$-16t^2 + 200 \tan 12^\circ = 0$$

$$t = \sqrt{\frac{200 \tan 12^\circ}{16}} \approx 1.63 \text{ sec}$$

MAX HEIGHT...

$$-32t + v_0 \sin 12^\circ = 0$$

$$\Rightarrow t = \frac{v_0 \sin 12^\circ}{32} \approx 0.815 \text{ sec}$$

WITH THIS TIME...

$$v_0 = \frac{200}{t \cos 12^\circ} \approx 125.44 \text{ FT/SEC}$$

$$-16t^2 + v_0 \sin 12^\circ t \approx 10.63 \text{ FT}$$