Math 233 - Test 1
September 11, 2025

Name key

Score ______

Show all work to receive full credit. Supply explanations where necessary.

1. (4 points) The vector $\vec{v}$ has initial point $(-2, 5)$ and terminal point $(3, -1)$. Find the component form of the unit vector with the direction of $\vec{v}$.

$\vec{V} = 5\hat{\imath} - 6\hat{\jmath}$

$\|\vec{V}\| = \sqrt{25+36} = \sqrt{61}$

$\dfrac{\vec{V}}{\|\vec{V}\|} = \boxed{\dfrac{5}{\sqrt{61}}\hat{\imath} - \dfrac{6}{\sqrt{61}}\hat{\jmath}}$

2. (9 points) Three forces in the xy-plane act on an object. Two of the forces have magnitudes 58 and 27, and they make angles of 53° and 152° with the positive x-axis, respectively. The third force acts in such a way that the resultant force (the sum of all three) is zero. Find the component form of the third force. Then determine the angle that it makes with the positive x-axis? Round to two decimal places.

$\vec{F}_1 = 58\cos 53°\,\hat{\imath} + 58\sin 53°\,\hat{\jmath} \approx 34.90527\hat{\imath} + 46.32086\hat{\jmath}$

$\vec{F}_2 = 27\cos 152°\,\hat{\imath} + 27\sin 152°\,\hat{\jmath} \approx -23.83959\hat{\imath} + 12.67573\hat{\jmath}$

$\vec{F}_3 = -(\vec{F}_1 + \vec{F}_2) \approx \boxed{-11.07\hat{\imath} - 59.00\hat{\jmath}}$

BEFORE ROUNDING TO TWO PLACES.

$\vec{F}_3$ IS IN QUAD III

WITH REFERENCE ANGLE $\approx \tan^{-1}\left(\dfrac{-58.99659178}{-11.06568634}\right) \approx 79.38°$

So, OUR ANGLE IS $\approx 259.38°$

3. (4 points) Let $\vec{u} = 4\hat{\imath} - 9\hat{\jmath}$. Find a vector in the xy-plane that has magnitude 2 and is perpendicular to $\vec{u}$.

SLOPE OF $\vec{u} = \dfrac{-9}{4}$

LET $\vec{V} = 9\hat{\imath} + 4\hat{\jmath}$.

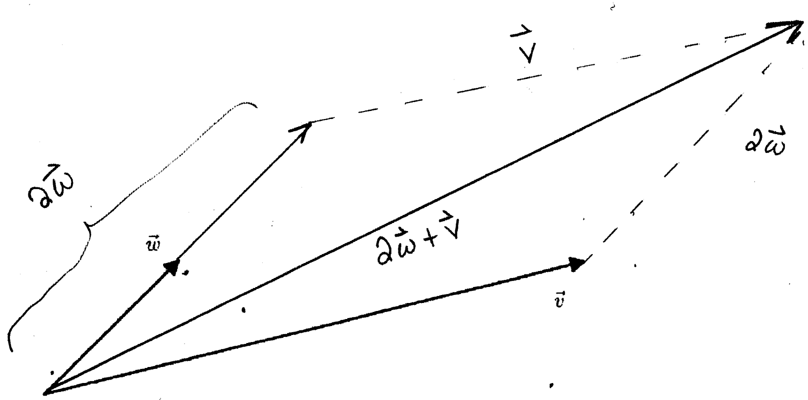
SLOPE OF $\vec{V} = \dfrac{4}{9}$,

SO $\vec{u} \perp \vec{V}$.

$\|\vec{V}\| = \sqrt{81+16} = \sqrt{97}$

$\dfrac{2\vec{V}}{\|\vec{V}\|} = \boxed{\dfrac{18}{\sqrt{97}}\hat{\imath} + \dfrac{8}{\sqrt{97}}\hat{\jmath}}$

4. (4 points) Referring to the vectors shown below, sketch the vector $2\vec{w} + \vec{v}$.



5. (5 points) Show that $A\,(5, 3, -1)$, $B\,(-5, -3, 1)$, and $C\,(-15, -9, 3)$ are collinear points.

$\vec{AB} = -10\hat{i} - 6\hat{j} + 2\hat{k}$

$\vec{AC} = -20\hat{i} - 12\hat{j} + 4\hat{k}$

$\frac{1}{2}\vec{AC} = \vec{AB}$

$\Rightarrow$ $\vec{AC}$ AND $\vec{AB}$ ARE PARALLEL

$\Rightarrow$ A, B, C LIE ALONG A LINE.

6. (5 points) Let M be the midpoint of $P\,(1, 4, -5)$ and $Q\,(9, -2, -1)$. Find the point M and compute $\|\vec{MQ}\|$.

$M = \left(\frac{1+9}{2}, \frac{4+(-2)}{2}, \frac{-5+(-1)}{2}\right)$

$\boxed{M = (5, 1, -3)}$

$\vec{MQ} = \langle 4, -3, 2 \rangle$

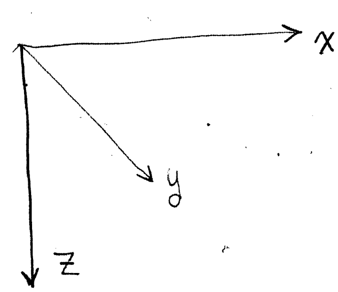
$\|\vec{MQ}\| = \sqrt{16+9+4}$

$= \boxed{\sqrt{29}}$

WHICH IS ONE-HALF OF $\|\vec{PQ}\|$.

7. (2 points) In a 3D coordinate system, suppose the positive x-axis points to the right on this page and the positive z-axis points down (on the page). Describe the placement of the positive y-axis.

POSITIVE y-AXIS POINTS OUT OF PAGE (TOWARD THE READER)



8. (6 points) Find the measure of the angle between $\vec{x} = -3\hat{\imath}+5\hat{\jmath}+2\hat{k}$ and $\vec{y} = 6\hat{\imath}-2\hat{\jmath}+\hat{k}$. Write your final answer in degrees rounded to the nearest tenth.

$$\vec{x}\cdot\vec{y} = \|\vec{x}\|\,\|\vec{y}\|\cos\theta$$

$$\vec{x}\cdot\vec{y} = -18-10+2 = -26$$

$$\|\vec{x}\| = \sqrt{9+25+4} = \sqrt{38}$$

$$\|\vec{y}\| = \sqrt{36+4+1} = \sqrt{41}$$

$$\theta = \cos^{-1}\left(\frac{-26}{\sqrt{38}\sqrt{41}}\right)$$

$$\approx \boxed{131.2^\circ}$$

9. (6 points) Find the projection of $\vec{p} = 4\hat{\imath}+5\hat{\jmath}-6\hat{k}$ onto $\vec{q} = -3\hat{\imath}+3\hat{\jmath}+2\hat{k}$.

$$\text{proj}_{\vec{q}}\,\vec{p} = \frac{\vec{p}\cdot\vec{q}}{\vec{q}\cdot\vec{q}}\,\vec{q}$$

$$= \frac{-12+15-12}{9+9+4}\,\vec{q} = \frac{-9}{22}\,\vec{q}$$

$$= -\frac{9}{22}\left(-3\hat{\imath}+3\hat{\jmath}+2\hat{k}\right)$$

$$= \boxed{\frac{27}{22}\hat{\imath} - \frac{27}{22}\hat{\jmath} - \frac{18}{22}\hat{k}}$$

10. (3 points) Assuming $\vec{a} \neq \vec{b}$, describe a case in which $\text{proj}_{\vec{a}}\,\vec{b} = \text{proj}_{\vec{b}}\,\vec{a}$. Explain.

$$\frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\,\vec{a} \qquad \frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\,\vec{b}$$

THEY'LL BOTH BE $\vec{0}$ WHEN $\vec{a}$ & $\vec{b}$ ARE ORTHOG.

11. (3 points) What can be said about the sign of $\vec{u}\cdot\vec{w}$ in each case below?

(a) The angle between $\vec{u}$ and $\vec{w}$ is obtuse.

NEGATIVE

(b) The angle between $\vec{u}$ and $\vec{w}$ is acute.

POSITIVE

(c) The angle between $\vec{u}$ and $\vec{w}$ is a right angle.

ZERO

12. (6 points) Let $\vec{v} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ and $\vec{w} = -3\hat{\imath} + 5\hat{k}$. Show that $\vec{v}$ is orthogonal to $\vec{v} \times \vec{w}$.

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 3 & -2 \\ -3 & 0 & 5 \end{vmatrix} = \hat{\imath}(15) - \hat{\jmath}(-1) + \hat{k}(9)$$
$$= 15\hat{\imath} + \hat{\jmath} + 9\hat{k}$$

$\vec{v} \cdot (\vec{v} \times \vec{w}) = (1)(15) + 3(1) - 2(9) = 0 \Rightarrow \vec{v}$ IS ORTHOG TO $\vec{v} \times \vec{w}$.

INCIDENTALLY,

$\vec{w} \cdot (\vec{v} \times \vec{w}) = -3(15) + (0)(1) + 5(9) = 0 \Rightarrow \vec{w}$ IS ALSO ORTHOG TO $\vec{v} \times \vec{w}$.

13. (7 points) Find the area of the triangle with vertices $P(-4, 3, 2)$, $Q(-1, -8, 7)$ and $R(3, 0, -2)$.

AREA $= \frac{1}{2} \| \vec{PQ} \times \vec{PR} \|$, $\vec{PQ} = \langle 3, -11, 5 \rangle$, $\vec{PR} = \langle 7, -3, -4 \rangle$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -11 & 5 \\ 7 & -3 & -4 \end{vmatrix}$$
$$= \hat{\imath}(59) - \hat{\jmath}(-47) + \hat{k}(68)$$
$$= 59\hat{\imath} + 47\hat{\jmath} + 68\hat{k}$$

$$\| \vec{PQ} \times \vec{PR} \| = \sqrt{(59)^2 + (47)^2 + (68)^2}$$
$$= \sqrt{10314}$$

$$\boxed{\text{AREA} = \frac{\sqrt{10314}}{2} \approx 50.78 \text{ unit}^2}$$

14. (8 points) Find parametric and symmetric equations of any line in the plane given by $x - 7y + 3z = 12$.

START BY GETTING TWO POINTS ON THE PLANE.

I'LL USE $(12, 0, 0)$ AND $(0, 0, 4)$.

$\qquad P \qquad\qquad Q$

DIRECTION

$= \vec{v} = \vec{PQ} = -12\hat{\imath} + 4\hat{k}$

USING $\vec{v}$ AND P ...

PARAMETRIC

$x = -12t + 12$

$y = 0$

$z = 4t$

SYMMETRIC

$\frac{x - 12}{-12} = \frac{z}{4}$, $y = 0$

15. (8 points) Find an equation of the plane that passes through $P(5,-2,8)$ and contains the line

$$\frac{x-4}{3}=y+3=\frac{z-1}{6}.$$

$Q(4,-3,1)$ IS ON THE LINE AND PLANE

$P(5,-2,8)$ IS ON PLANE

$\vec{PQ}=\langle 1,1,7\rangle$ IS ON PLANE

$\vec{v}=\langle 3,1,6\rangle$ IS ALSO ON PLANE

$\vec{n}=\vec{PQ}\times\vec{v}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&7\\3&1&6\end{vmatrix}=-\hat{i}+15\hat{j}-2\hat{k}$

I'LL USE $\vec{n}=\hat{i}-15\hat{j}+2\hat{k}$

USING $\vec{n}$ & P...

$x-15y+2z=5+30+16$

$\boxed{x-15y+2z=51}$

16. (8 points) A plane is described by the equation $5x-3y+2z=2$.

(a) Show that $Q(2,4,3)$ is NOT a point on the plane.

$5(2)-3(4)+2(3)=4\neq 2$

(b) Find the distance from the point Q to the plane.

$$\text{DISTANCE}=\frac{\text{DIFFERENCE OF } d\text{'s}}{\sqrt{5^2+(-3)^2+2^2}}=\frac{4-2}{\sqrt{38}}=\boxed{\frac{2}{\sqrt{38}}}$$

(c) Now consider the point $R(-1,-2,5)$. Without actually computing the distance from R to the plane, say which point, Q or R, is closer to the plane. How do you know?

$5(-1)-3(-2)+2(5)=11$ ← GREATER THAN 4 FROM (a) MEANS R IS FARTHER FROM PLANE!

$\boxed{\text{Q IS CLOSER}}$

Common point? Let $x=0$ $\longrightarrow$ $y-5z=4$, $-y-3z=1$; $-8z=5$ $\longrightarrow$ $z=\frac{-5}{8}$, $y=4+5\left(\frac{-5}{8}\right)=\frac{7}{8}$

$\left(0, \frac{7}{8}, -\frac{5}{8}\right)$

17. (8 points) Find parametric equations for the line of intersection of the two planes.

$$P_1: \quad x+y-5z=4 \qquad\qquad P_2: \quad 2x-y-3z=1$$

$\vec{n}_1=\hat{i}+\hat{j}-5\hat{k}$ $\qquad$ $\vec{n}_2=2\hat{i}-\hat{j}-3\hat{k}$

$$\vec{V}=\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & -5\\ 2 & -1 & -3\end{vmatrix}$$

$$=\hat{i}(-8)-\hat{j}(7)+\hat{k}(-3)$$

$$=-8\hat{i}-7\hat{j}-3\hat{k}$$

I'll use $8\hat{i}+7\hat{j}+3\hat{k}$

$$\boxed{\begin{aligned}x&=8t\\ y&=7t+\tfrac{7}{8}\\ z&=3t-\tfrac{5}{8}\end{aligned}}$$

18. (4 points) Find the measure of the angle that $\vec{w}=5\hat{\imath}-7\hat{\jmath}+9\hat{k}$ makes with the positive y-axis. Write your final answer in degrees rounded to the nearest tenth.

$\|\vec{w}\|=\sqrt{25+49+81}=\sqrt{155}$

$$\vec{w}\cdot\hat{j}=\|\vec{w}\|\,\|\hat{j}\|\cos\theta$$

$$-7=\sqrt{155}\,\sqrt{1}\,\cos\theta$$

$$\cos\theta=\frac{-7}{\sqrt{155}}$$

$$\Rightarrow \boxed{\theta\approx 124.2^\circ}$$