Math 233 - Test 1

Name _____

September 11, 2025

Score _____

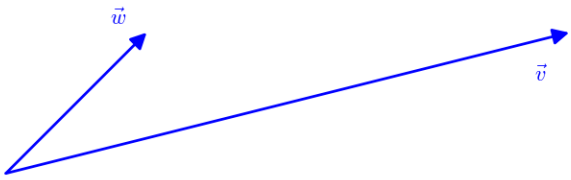
Show all work to receive full credit. Supply explanations where necessary.

1. (4 points) The vector $\vec{v}$ has initial point $(-2, 5)$ and terminal point $(3, -1)$. Find the component form of the unit vector with the direction of $\vec{v}$.

2. (9 points) Three forces in the xy-plane act on an object. Two of the forces have magnitudes 58 and 27, and they make angles of $53°$ and $152°$ with the positive x-axis, respectively. The third force acts in such a way that the resultant force (the sum of all three) is zero. Find the component form of the third force. Then determine the angle that it makes with the positive x-axis? Round to two decimal places.

3. (4 points) Let $\vec{u} = 4\hat{\imath} - 9\vec{j}$. Find a vector in the xy-plane that has magnitude 2 and is perpendicular to $\vec{u}$.

4. (4 points) Referring to the vectors shown below, sketch the vector $2\vec{w} + \vec{v}$.



5. (5 points) Show that $A\,(5, 3, -1)$, $B\,(-5, -3, 1)$, and $C\,(-15, -9, 3)$ are collinear points.

6. (5 points) Let M be the midpoint of $P\,(1, 4, -5)$ and $Q\,(9, -2, -1)$. Find the point M and compute $\|\vec{MQ}\|$.

7. (2 points) In a 3D coordinate system, suppose the positive x-axis points to the right on this page and the positive z-axis points down (on the page). Describe the placement of the positive y-axis.

8. (6 points) Find the measure of the angle between $\vec{x} = -3\hat{\imath} + 5\hat{\jmath} + 2\hat{k}$ and $\vec{y} = 6\hat{\imath} - 2\hat{\jmath} + \hat{k}$. Write your final answer in degrees rounded to the nearest tenth.

9. (6 points) Find the projection of $\vec{p} = 4\hat{\imath} + 5\hat{\jmath} - 6\hat{k}$ onto $\vec{q} = -3\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$.

10. (3 points) Assuming $\vec{a} \neq \vec{b}$, describe a case in which $\text{proj}_{\vec{a}}\,\vec{b} = \text{proj}_{\vec{b}}\,\vec{a}$. Explain.

11. (3 points) What can be said about the sign of $\vec{u} \cdot \vec{w}$ in each case below?

    (a) The angle between $\vec{u}$ and $\vec{w}$ is obtuse.

    (b) The angle between $\vec{u}$ and $\vec{w}$ is acute.

    (c) The angle between $\vec{u}$ and $\vec{w}$ is a right angle.

12. (6 points) Let $\vec{v} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ and $\vec{w} = -3\hat{\imath} + 5\hat{k}$. Show that $\vec{v}$ is orthogonal to $\vec{v} \times \vec{w}$.

13. (7 points) Find the area of the triangle with vertices $P\,(-4, 3, 2)$, $Q\,(-1, -8, 7)$ and $R\,(3, 0, -2)$.

14. (8 points) Find parametric and symmetric equations of any line in the plane given by $x - 7y + 3z = 12$.

15. (8 points) Find an equation of the plane that passes through $P\,(5,-2,8)$ and contains the line

$$\frac{x-4}{3} = y+3 = \frac{z-1}{6}.$$

16. (8 points) A plane is described by the equation $5x - 3y + 2z = 2$.

(a) Show that $Q\,(2,4,3)$ is NOT a point on the plane.

(b) Find the distance from the point Q to the plane.

(c) Now consider the point $R\,(-1,-2,5)$. Without actually computing the distance from R to the plane, say which point, Q or R, is closer to the plane. How do you know?

17. (8 points) Find parametric equations for the line of intersection of the two planes.

$$P_1: \quad x + y - 5z = 4 \qquad\qquad P_2: \quad 2x - y - 3z = 1$$

18. (4 points) Find the measure of the angle that $\vec{w} = 5\hat{\imath} - 7\hat{\jmath} + 9\hat{k}$ makes with the positive y-axis. Write your final answer in degrees rounded to the nearest tenth.