Math 233 - Test 2

October 9, 2025

Name ______________________

Score __________

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) The velocity vector of a moving particle is given by

$$\vec{v}(t) = (6t^2 + 2t)\,\hat{\imath} + (8\sin 2t)\,\hat{\jmath} + 3e^{-t}\,\hat{k}.$$

   Find the position vector if the particle's motion began (at $t = 0$) at the point $(4, 9, 12)$.

2. (6 points) For $t \geq 0$, let $\vec{r}(t) = (\sin t - t\,\cos t)\,\hat{\imath} + (\cos t + t\,\sin t)\,\hat{\jmath}$. Compute the principal unit tangent vector, $\hat{T}(t)$.

3. (8 points) A curve in the xy-plane is described by the following parametric equations. Find the curvature function, $\kappa(t)$.

$$x = \frac{t^2}{2}, \quad y = \frac{t^3}{3}$$

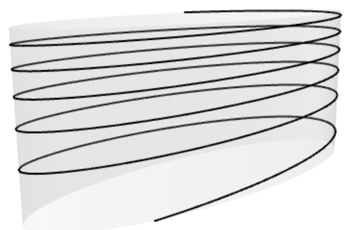
4. (12 points) Let $\vec{r}(t) = t\,\hat{\imath} - \sin 2t\,\hat{\jmath} + \cos 2t\,\hat{k}$. Starting from $t = 0$, find the arc-length parameter, $s(t)$, and then reparameterize $\vec{r}$ in terms of s.

Follow-up: Show that when the function is reparameterized, its derivative has magnitude 1.

5. (8 points) A wire is wrapped around an elliptical steel tube so that the wire has the shape of the graph of

$$\vec{r}(t) = 6\cos(t)\,\hat{\imath} + 2\sin(t)\,\hat{\jmath} + \sqrt{t}\,\hat{k}, \quad 1 \le t \le 36,$$

where $\vec{r}$ is in centimeters. Set up the definite integral that gives the length of the wire. Use your calculator to approximate the value of your integral.



6. (10 points) Let $\vec{r}(t) = (2t+3)\,\hat{\imath} + (t^2-1)\,\hat{\jmath}$. Compute the tangential and normal components of acceleration.

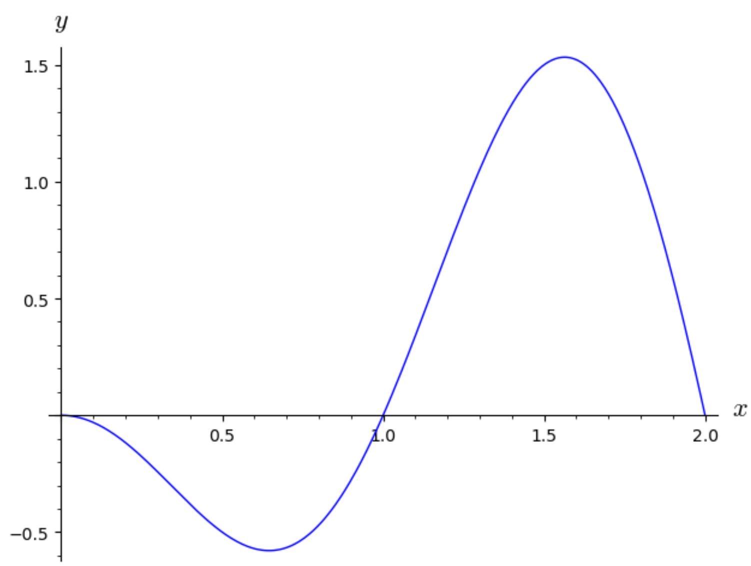
7. (8 points) A baseball, hit 3 feet above the ground, leaves the bat at an angle of 45° and is caught by an outfielder at a height of 3 feet above the ground and 300 feet from home plate. What is the initial speed of the ball? (To receive full credit, you must write and use the vector-valued function $\vec{r}(t)$ that gives the position of the ball at time t. Also ignore air resistance and use $g \approx 32\,\mathrm{ft\,s^{-2}}$.)

8. (6 points) Suppose a particle moves along the given curve from **right to left**. Sketch and label each of the following. Make note of the scale.

   (a) The principal unit tangent vector at the point of greatest curvature.

   (b) A point where the principal unit normal vector does not exist.

   (c) The principal unit normal vector at the point where $x = 0.5$.



9. (2 points) Sketch, or describe in detail, a 2-dimensional curve whose curvature is constant and nonzero. Then say what the curvature of your curve actually is.

10. (8 points) Let $\vec{r}(t) = -\cos 3t\,\hat{\imath} - \sin 3t\,\hat{\jmath} + 4t\,\hat{k}$. Compute $\hat{N}(t)$.

11. (8 points) Consider the function $f(x, y) = \ln(4 - x - y)$.

    (a) Evaluate $f(2, 1)$.

    (b) What is the domain of f?

    (c) What is the range of f?

    (d) Sketch the level curve $f(x, y) = 0$.

    (e) Sketch the level curve $f(x, y) = 1$.

12. (8 points) Let $G(x,y,z) = \sqrt{2x-3y+z}$.

    (a) Compute $G(-1,-2,4)$.

    (b) What is the domain of G?

    (c) What is the range of G?

    (d) Describe, in detail, the level surface $G(x,y,z) = 2$.

13. (8 points) Describe the **surface in space** that is defined by each equation.

    (a) $x^2 + y^2 = 4$

    (b) $z = \sqrt{16 - x^2 - y^2}$

    (c) $y = x^2$

    (d) $z = x + y$