Math 233 - Final Exam B

Name ____________________________

December 11, 2025

Score ___________

Show all work to receive full credit. Supply explanations where necessary.

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1. (10 points) Let $\vec{r}(t) = (8 - t^2)\,\hat{\imath} + t^3\,\hat{\jmath} + \frac{\sqrt{5}}{2}t^2\,\hat{k}$. Starting from $t = 0$, find the arc-length parameter $s(t)$, and then use the arc-length parameter to find the length of the graph of $\vec{r}$ over the interval from $t = 0$ to $t = 7$.

2. (10 points) In 1988, East German Petra Felke set a women's world record by throwing a javelin 262.4 ft (over 80 m!). Felke launched the javelin from 6.5 ft above the ground at 40° angle with the horizontal. What was the javelin's initial speed? And when did it reach it's maximum height? (To receive full credit, you must write and use the vector-valued function $\vec{r}(t)$ that gives the position of the object at time t. Also ignore air resistance and use $g \approx 32\,\mathrm{ft\,s^{-2}}$.)

3. (10 points) The total resistance, R, of two resistors connected in parallel satisfies the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2},$$

where R_1 and R_2 are the resistances of the connected resistors. Determine the total differential dR. Then use differentials to approximate ΔR as (R_1, R_2) changes from $(10, 15)$ to $(10.4, 14.7)$.

4. (10 points) Consider the surface described by the equation $\sqrt{\dfrac{z+x}{y-1}} = z^2$. (Advice: This problem will be easier if you start by doing some algebra to rewrite the equation.)

   (a) Find an equation of the plane tangent to the surface at the point $(3, 5, 1)$.

   (b) Find symmetric equations for the line normal to the surface at $(3, 5, 1)$.

5. (10 points) The region R is the 1st quadrant region bounded by the graphs of $y = 2x$ and $y = \frac{1}{2}x^2$. Sketch the region of integration, write the double integral as an iterated integral with both orders of integration, and evaluate either one of your iterated integrals.

$$\iint\limits_R (2x^2 + 8y)\, dA$$

6. (10 points) Use Lagrange multipliers to find the minimum and maximum values of $F(x, y, z) = 2x + y - 2z$ on the sphere $x^2 + y^2 + z^2 = 4$.

7. (10 points) Evaluate $\int_C \frac{x}{1+y^2}\, ds$, where the curve C is made up of two line segments: the first from $(1, 0)$ to $(3, 1)$, and the second from $(3, 1)$ to $(5, 1)$.