

Math 233 - Quiz 1

August 20, 2026

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due August 25.

1. (4 points) Let $\vec{w}$ be the vector from $A(3, -5)$ to $B(6, 1)$.

- (a) Find the component form of $\vec{w}$.

$$\vec{w} = \vec{AB} = \langle 6-3, 1-(-5) \rangle = \langle 3, 6 \rangle = 3\hat{i} + 6\hat{j}$$

- (b) Compute $\|\vec{w}\|$.

$$\|\vec{w}\| = \sqrt{(3)^2 + (6)^2} = \sqrt{45} = 3\sqrt{5}$$

- (c) Determine a vector of magnitude 10 with the same direction as $\vec{w}$.

$$\frac{10\vec{w}}{\|\vec{w}\|} = \frac{10(3\hat{i} + 6\hat{j})}{3\sqrt{5}} = \frac{10\hat{i} + 20\hat{j}}{\sqrt{5}}$$

- (d) What angle does $\vec{w}$ make with the positive x -axis? Write your answer in degrees, rounded to the nearest tenth. (Do not use the dot product to solve this problem.)

$$\vec{w} \text{ IS IN THE 1ST QUAD AND } \tan \theta = \frac{6}{3} = 2$$

$$\theta = \tan^{-1}(2) \approx 63.4^\circ$$

2. (2 points) Let $\vec{u} = 4\hat{i} - 7\hat{j}$ and let $\vec{v}$ be the 2D vector of magnitude 5 that makes a 225° angle with the positive x -axis. Compute $2\vec{u} - \vec{v}$ and then find its magnitude.

$$\vec{u} = 4\hat{i} - 7\hat{j}$$

$$\vec{v} = 5\cos 225^\circ \hat{i} + 5\sin 225^\circ \hat{j}$$

$$= -\frac{5\sqrt{2}}{2}\hat{i} - \frac{5\sqrt{2}}{2}\hat{j}$$

$$2\vec{u} - \vec{v} = \left(8 + \frac{5\sqrt{2}}{2}\right)\hat{i} + \left(-14 + \frac{5\sqrt{2}}{2}\right)\hat{j}$$

$$= \frac{16+5\sqrt{2}}{2}\hat{i} + \frac{5\sqrt{2}-28}{2}\hat{j} \approx 11.536\hat{i} - 10.464\hat{j}$$

$$\|2\vec{u} - \vec{v}\| = \sqrt{\left(\frac{16+5\sqrt{2}}{2}\right)^2 + \left(\frac{5\sqrt{2}-28}{2}\right)^2}$$

$$\approx \sqrt{242.57...}$$

Turn over.

$$\approx 15.575$$

3. (1 point) Use vectors to show that the points $P(8, -9, 8)$, $Q(2, 1, 0)$, and $R(-10, 21, -16)$ are collinear.

$$\vec{PQ} = -6\hat{i} + 10\hat{j} - 8\hat{k}$$

$$\vec{PR} = -18\hat{i} + 30\hat{j} - 24\hat{k}$$

SINCE $\vec{PR} = 3\vec{PQ}$, $\vec{PR}$ AND $\vec{PQ}$ ARE PARALLEL.

PARALLEL & SHARING POINT P

$\Rightarrow$ P, Q, R LIE ALONG THE SAME LINE.

4. (1 point) Write an equation for the plane passing through $(5, 8, -7)$ that is parallel to the xy -plane.

x, y ARBITRARY

$$z = -7$$

5. (2 points) Let $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and let $\vec{b} = \vec{v} - \vec{w}$, where $\vec{v} = 2\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{w} = 6\hat{i} + \hat{j} + 2\hat{k}$. Find the measure of the angle between $\vec{a}$ and $\vec{b}$. Write your answer in radians, rounded to the nearest hundredth.

$$\vec{b} = \vec{v} - \vec{w} = (2-6)\hat{i} + (1-1)\hat{j} + (4-2)\hat{k} = -4\hat{i} + 2\hat{k}$$

$$\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} = \frac{-12 + 4}{\sqrt{14} \sqrt{20}} = \frac{-8}{\sqrt{280}} = -\frac{4}{\sqrt{70}}$$

$$\theta = \cos^{-1}\left(-\frac{4}{\sqrt{70}}\right) \approx \boxed{2.07}$$

$$\approx 118.56^\circ$$