Math 233 - Quiz 2

August 27, 2026

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Find a unit vector that is orthogonal to $\vec{w} = 2\hat{\imath} + 8\hat{\jmath} - 7\hat{k}$.

$\vec{u} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

$\vec{u} \cdot \vec{w} = 0 \Rightarrow 2a + 8b - 7c = 0$

Let's use $a = 2, b = 3, c = 4$

$\vec{u} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$

$\|\vec{u}\| = \sqrt{4+9+16} = \sqrt{29}$

$\frac{\vec{u}}{\|\vec{u}\|} = \frac{1}{\sqrt{29}}(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$

2. (3 points) Find the measure of the angle at vertex B in triangle ABC. Write your answer in degrees, rounded to the nearest tenth.

$A(1,1,3), \quad B(3,6,5), \quad C(-1,-2,5)$

$\vec{BA} = -2\hat{\imath} - 5\hat{\jmath} - 2\hat{k}$

$\vec{BC} = -4\hat{\imath} - 8\hat{\jmath}$

$$\cos\theta = \frac{\vec{BA} \cdot \vec{BC}}{\|\vec{BA}\| \|\vec{BC}\|} = \frac{8+40}{\sqrt{4+25+4}\sqrt{16+64}} = \frac{48}{\sqrt{33}\sqrt{80}} \Rightarrow \theta = \cos^{-1}\left(\frac{48}{\sqrt{33}\sqrt{80}}\right) \approx 20.9^\circ$$

3. (1 point) Find the projection of $\vec{b}$ onto $\vec{a}$, where $\vec{a} = -\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{b} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$.

$$\text{proj}_{\vec{a}}\,\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a} = \frac{-2-3-2}{1+9+1}\vec{a} = \frac{-7}{11}(-\hat{\imath} + 3\hat{\jmath} - \hat{k})$$

or $\frac{7}{11}\hat{\imath} - \frac{21}{11}\hat{\jmath} + \frac{7}{11}\hat{k}$

4. (2 points) Find the measure of the angle that $\vec{s} = 5\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ makes with the positive x-axis.

Angle between $\vec{s}$ & $\hat{\imath}$:

$$\cos\theta = \frac{\vec{s}\cdot\hat{\imath}}{\|\vec{s}\|\|\hat{\imath}\|} = \frac{5}{\sqrt{25+4+16}} = \frac{5}{\sqrt{45}} = \frac{5}{3\sqrt{5}} = \frac{\sqrt{5}}{3}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{5}}{3}\right) \approx 41.8^\circ$$

5. (2 points) Let $\vec{u} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{w} = 2\hat{\imath} - 3\hat{\jmath} - 4\hat{k}$. Compute $\vec{w} \times \vec{u}$.

$$\vec{w} \times \vec{u} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -3 & -4 \\ 1 & 2 & 3 \end{vmatrix} = \hat{\imath}(-9+8) - \hat{\jmath}(6+4) + \hat{k}(4+3)$$

$$= -\hat{\imath} - 10\hat{\jmath} + 7\hat{k}$$