

Math 233 - Quiz 5

September 24, 2026

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (6 points) Let $\vec{r}(t) = (t^2 - t)\hat{i} + \frac{1}{6}(4t - 1)^{3/2}\hat{j} + 5\hat{k}$. Compute the arc-length parameter s starting from $t = 1$. Then solve for t in terms of s .

$$\vec{r}'(t) = (2t-1)\hat{i} + (4t-1)^{1/2}\hat{j}$$

$$\|\vec{r}'(t)\| = \sqrt{(2t-1)^2 + (4t-1)} = \sqrt{4t^2} = 2t, \text{ assuming } t > 0.$$

$$s(t) = \int_1^t 2\tau \, d\tau = \tau^2 \Big|_1^t = t^2 - 1$$

$$s(t) = t^2 - 1$$

$$t(s) = \sqrt{s+1}$$

2. (4 points) Let $\vec{r}(t) = -\cos 3t\hat{i} - \sin 3t\hat{j} + 4t\hat{k}$. Compute the curvature function $\kappa(t)$.

$$\vec{r}'(t) = 3\sin 3t\hat{i} - 3\cos 3t\hat{j} + 4\hat{k}$$

$$\|\vec{r}'(t)\| = \sqrt{9\sin^2 3t + 9\cos^2 3t + 16}$$

$$= \sqrt{25} = 5$$

$$\kappa(t) = \frac{\|\hat{T}'(t)\|}{\|\vec{r}'(t)\|} = \frac{9/5}{5}$$

$$\hat{T}(t) = \frac{3}{5}\sin 3t\hat{i} - \frac{3}{5}\cos 3t\hat{j} + \frac{4}{5}\hat{k}$$

$$= \frac{9}{25}$$

$$\hat{T}'(t) = \frac{9}{5}\cos 3t\hat{i} + \frac{9}{5}\sin 3t\hat{j}$$

$$\|\hat{T}'(t)\| = \frac{9}{5}$$

$$\kappa(t) = \frac{9}{25}$$