

Math 233 - Quiz 6

October 1, 2026

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) What are the three important characteristics of the principal unit normal vector.

- IT POINTS IN THE DIRECTION $\hat{T}$ IS CHANGING (INTO THE CURVE)
- IT'S A UNIT VECTOR.
- IT'S NORMAL TO THE CURVE.

2. (4 points) Let $\vec{r}(t) = \cos 3t \hat{i} - \sin 3t \hat{j} - 4t \hat{k}$. Compute the principal unit normal vector $\hat{N}(t)$.

$$\vec{r}'(t) = -3 \sin 3t \hat{i} - 3 \cos 3t \hat{j} - 4 \hat{k}$$

$$\|\vec{r}'(t)\| = \sqrt{9 + 16} = 5$$

$$\hat{T}(t) = -\frac{3}{5} \sin 3t \hat{i} - \frac{3}{5} \cos 3t \hat{j} - \frac{4}{5} \hat{k}$$

$$\hat{T}'(t) = -\frac{9}{5} \cos 3t \hat{i} + \frac{9}{5} \sin 3t \hat{j}$$

MAGNITUDE = $\frac{9}{5}$

$$\hat{N}(t) = -\cos 3t \hat{i} + \sin 3t \hat{j}$$

3. (4 points) Each of these equations defines a surface in 3-space. Briefly describe each surface.

(a) $10 + 3x^2 - 8y^2 + 5z^2 = 0$ $3x^2 - 8y^2 + 5z^2 = -10$

Cross sections in x, y, z order: Hyp., Ellipse, Hyp.

Ellipses, $3x^2 + 5z^2 = -10 + 8y^2$, DNE for small y.

Hyperboloid
of Two Sheets

(b) $4x^2 - \frac{1}{4}y^2 - \frac{1}{9}z^2 = -1$

Cross sections in x, y, z order: Ellipse, Hyp, Hyp.

Ellipses, $\frac{1}{4}y^2 + \frac{1}{9}z^2 = 4x^2 + 1$, exist for all x.

Hyperboloid
of One Sheet

(c) $z = (x+1)^2 + 1$

y is arbitrary.

Parabolic cylinder

Generating curve is a
parabola in xz-plane.

Rulings are parallel to $\hat{j}$.

(d) $4x^2 + 4y^2 - z = 1$

Cross sections in x, y, z order: Parabola, Parabola, Circle

Circular paraboloid

Opens up z-axis with
vertex at $(0, 0, -1)$.