Math 233 - Homework 1

February 4, 2021

Name key

Score _____

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due on February 11.

1. (2 points) Find a vector of magnitude 10 that has the direction of $\vec{v} = \langle 7, -1, 3 \rangle$.

$$\frac{10\vec{v}}{\|\vec{v}\|} = \frac{10}{\sqrt{59}} \langle 7, -1, 3 \rangle = \boxed{\frac{70\hat{i} - 10\hat{j} + 30\hat{k}}{\sqrt{59}}}$$

$$\|\vec{v}\| = \sqrt{(7)^2 + (-1)^2 + (3)^2}$$

$$= \sqrt{59}$$

2. (2 points) Let $\vec{u} = 3\hat{i} + 2\hat{k}$ and $\vec{v} = 2\hat{j} + 4\hat{k}$. Find the projection of $\vec{v}$ onto $\vec{u}$.

$$\text{proj}_{\vec{u}} \vec{v} = \frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}} \vec{u}$$

$$= \frac{0 + 0 + 8}{9 + 0 + 4} \vec{u} = \boxed{\frac{8}{13}\vec{u} = \frac{24}{13}\hat{i} + \frac{16}{13}\hat{k}}$$

3. (2 points) Find the distance between the point $A(-3, 1, 1)$ and the line with symmetric equations $x = -y = -z$.

$x = t$, $y = -t$, $z = -t$

Passes through $P(0,0,0)$ w/ direction $\hat{i} - \hat{j} - \hat{k}$

$\vec{PA} = -3\hat{i} + \hat{j} + \hat{k}$

$$\vec{PA} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 1 \\ 1 & -1 & -1 \end{vmatrix} = \hat{i}(0) - \hat{j}(2) + \hat{k}(2) = -2\hat{j} + 2\hat{k}$$

$$d = \frac{\|\vec{PA} \times \vec{v}\|}{\|\vec{v}\|} = \frac{\sqrt{8}}{\sqrt{3}} = \sqrt{\frac{8}{3}}$$

4. (2 points) Show that the line passing through the points $P(3, 1, 0)$ and $Q(1, 4, -3)$ is perpendicular to the line with parametric equations $x = 3t$, $y = 3+8t$, and $z = -7+6t$.

$\vec{PQ} = -2\hat{i} + 3\hat{j} - 3\hat{k}$

Using $P(3,1,0)$...

$x = 3 - 2s$

$y = 1 + 3s$

$z = -3s$

↑ Line through P & Q

Other line:

$x = 3t$

$y = 3 + 8t$

$z = -7 + 6t$

Direction $= \vec{u} = 3\hat{i} + 8\hat{j} + 6\hat{k}$

$\vec{u} \cdot \vec{PQ} = 3(-2) + 8(3) + 6(-3) = 0 \Rightarrow$ Directions are orthog,

Point of intersection?

$1 + 3s = 3 + 8t$

$-3s = -7 + 6t$

$1 = -4 + 14t$

$t = \frac{5}{14}$, $s = \frac{34}{21}$

These don't satisfy the x equations.

Lines do not intersect.

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5. (2 points) Rewrite the equation in standard form and identify the quadric surface.

(a) $6x = 3y^2 + 2z^2$

$$x = \frac{3y^2}{6} + \frac{2z^2}{6} \quad \text{or} \quad x = \frac{y^2}{\sqrt{2}^2} + \frac{z^2}{\sqrt{3}^2}$$

Elliptic paraboloid opening "up" the pos x-axis.

(b) $-x^2 + 36y^2 + 36z^2 = 9$

$$-\frac{x^2}{9} + 4y^2 + 4z^2 = 1$$

$$-\frac{x^2}{3^2} + \frac{y^2}{\left(\frac{1}{2}\right)^2} + \frac{z^2}{\left(\frac{1}{2}\right)^2} = 1$$

Hyperboloid of one sheet centered on x-axis.