Math 233 - Homework 3

March 4, 2021

Name key

Score _____

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due on March 11.

1. (1 point) Determine the domain of each function.

   (a) $f(x,y) = 4\ln(y^2 - x)$

   $y^2 - x > 0 \Rightarrow$ Domain $= \{(x,y) : x < y^2\}$

   (b) $z = \sqrt{100 - 4x^2 - 25y^2}$

   $100 - 4x^2 - 25y^2 \geq 0 \Rightarrow$ Domain $= \{(x,y) : 4x^2 + 25y^2 \leq 100\}$

2. (1 point) Determine the range of $g(x,y) = \sqrt{16 - 4x^2 - y^2}$.

   $g(x,y) = \sqrt{16 - (4x^2 + y^2)}$

   Subtracting something positive $\Rightarrow$ Greatest value of g is $\sqrt{16}$. Least value is 0.

   Range $= \{z : 0 \leq z \leq 4\}$

3. (1 point) Describe the level surface $w(x,y,z) = 4$ when $w(x,y,z) = x^2 + y^2 - z^2$.

   $x^2 + y^2 - z^2 = 4$

   Describes a hyperboloid of one sheet centered on the z-axis.

Turn over.

4. (2 points) Use algebraic techniques to find the limit: $\lim_{(x,y)\to(0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$

$$\lim_{(x,y)\to(0,0)} \frac{x(x-y)}{\sqrt{x}-\sqrt{y}} = \lim_{(x,y)\to(0,0)} \frac{x(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})}{\sqrt{x}-\sqrt{y}}$$

$$= \lim_{(x,y)\to(0,0)} x(\sqrt{x}+\sqrt{y}) = \boxed{0}$$

5. (2 points) Show that the limit does not exist: $\lim_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^4 + y^2}$

TWO PATH TEST...

$$y = 0: \quad \lim_{x\to 0} \frac{0}{x^4} = \lim_{x\to 0} 0 = 0$$

$$y = x^2: \quad \lim_{x\to 0} \frac{x^4}{x^4+x^4} = \lim_{x\to 0} \frac{x^4}{2x^4} = \lim_{x\to 0} \frac{1}{2} = \frac{1}{2}$$

TWO LIMITS ⇓ LIMIT DNE

6. (2 points) Let $z = x^2 + 3xy + 2y^2$. Find $\dfrac{\partial^2 z}{\partial x^2}$ and $\dfrac{\partial^2 z}{\partial y^2}$.

$$\frac{\partial z}{\partial x} = 2x + 3y \;\Rightarrow\; \boxed{\frac{\partial^2 z}{\partial x^2} = 2}$$

$$\frac{\partial z}{\partial y} = 3x + 4y \;\Rightarrow\; \boxed{\frac{\partial^2 z}{\partial y^2} = 4}$$