

**Math 233 - Homework 4**  
April 1, 2021

Name key  
Score \_\_\_\_\_

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due April 8.

1. (2 points) Find the directional derivative of  $f$  at  $P$  in the direction of  $\vec{v}$ .

$$f(x, y, z) = y^2 + xz, \quad P(1, 2, 2), \quad \vec{v} = \langle 2, -1, 2 \rangle.$$

$$\vec{\nabla} f(x, y, z) = z\hat{i} + 2y\hat{j} + x\hat{k}$$

$$\vec{\nabla} f(1, 2, 2) = 2\hat{i} + 4\hat{j} + \hat{k}$$

$$D_{\vec{v}} f(1, 2, 2) = \vec{\nabla} f(1, 2, 2) \cdot \frac{\vec{v}}{\|\vec{v}\|}$$

$$= \frac{4-4+2}{\sqrt{4+1+4}} = \frac{2}{\sqrt{9}} = \boxed{\frac{2}{3}}$$

2. (2 points) Find an equation of the plane tangent to the graph of  $z = x^2 - 2xy + y^2$  at the point  $P(1, 2, 1)$ .

$$\text{Let } F(x, y, z) = x^2 - 2xy + y^2 - z.$$

THE GIVEN SURFACE IS THE LEVEL SURFACE  $F(x, y, z) = 0$ .

$$\vec{\nabla} F(x, y, z) = (2x - 2y)\hat{i} + (-2x + 2y)\hat{j} - \hat{k}$$

$$\vec{n} = \vec{\nabla} F(1, 2, 1) = -2\hat{i} + 2\hat{j} - \hat{k}$$

TANGENT PLANE ...

$$-2(x-1) + 2(y-2) - (z-1) = 0$$

OR

$$-2x + 2y - z = 1$$

Turn over.

3. (2 points) Let  $z = f(x, y) = x^2 + 3xy - y^2$ . Use differentials to approximate  $\Delta z$  as  $x$  changes from 2.00 to 2.05 and  $y$  changes from 3.00 to 2.96.

$$dz = (2x+3y)dx + (3x-2y)dy$$

$$\Delta z \approx (2x+3y)\Delta x + (3x-2y)\Delta y \quad \Delta x = 0.05, \Delta y = -0.04$$

$$\Delta z \approx (2 \cdot 2 + 3 \cdot 3)(0.05) + (3 \cdot 2 - 2 \cdot 3)(-0.04)$$

$$= 13(0.05) + 0(-0.04) = \boxed{0.65}$$

4. (2 points) Find the critical points of  $f(x, y) = 15x^3 - 3xy + 15y^3$ .

$$f_x(x, y) = 45x^2 - 3y = 0$$

$$f_y(x, y) = -3x + 45y^2 = 0$$

$$\downarrow$$

$$45x^2 - 3y = 0$$

$$\underline{3xy - 45y^3 = 0}$$

$$\underline{45x^2 - 45y^3 = 0}$$

$$x^3 = y^3 \Rightarrow x = y$$

$$\downarrow$$

$$45x^2 - 3x = 0$$

$$\downarrow$$

$$3x(15x - 1) = 0$$

$$x = 0, x = \frac{1}{15}$$

Crit pts :

$$(0, 0),$$

$$(\frac{1}{15}, \frac{1}{15})$$

5. (2 points) Find and classify the critical points of  $f(x, y) = 9 - x^4y^4$ .

$$f_x(x, y) = -4x^3y^4 = 0$$

$$f_y(x, y) = -4x^4y^3 = 0$$

$$\left. \begin{array}{l} x = 0, y \text{ anything} \\ \text{or} \\ y = 0, x \text{ anything} \end{array} \right\}$$

Crit pts are all

ordered pairs

$(0, y)$  or  $(x, 0)$ .

$$D(x, y) = \det \begin{pmatrix} -12x^2y^4 & -16x^3y^3 \\ -16x^3y^3 & -12x^4y^2 \end{pmatrix}$$

$$= 144x^6y^6 - 256x^6y^6$$

$$= -112x^6y^6$$

$$D(x, y) = 0 \quad \Rightarrow \\ \text{AT ALL CRIT POINTS.}$$

Test inconclusive,

but all crit pts

give a MAX VALUE  
OF 9.

Obvious?