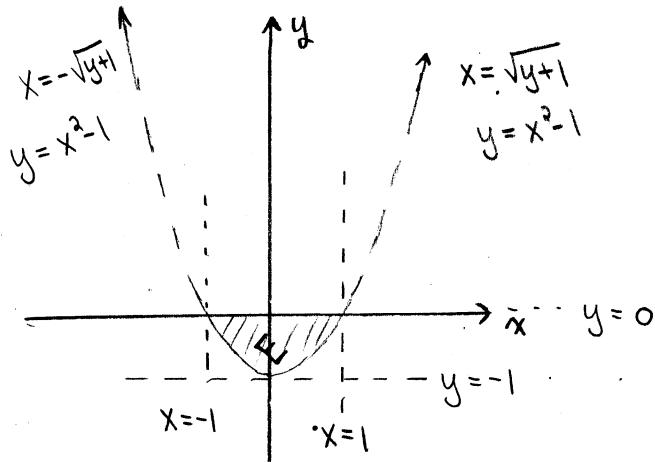


**Math 233 - Homework 5**  
April 29, 2021

Name key  
Score \_\_\_\_\_

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due May 6.

1. (2 points) Reverse the order of integration and evaluate.



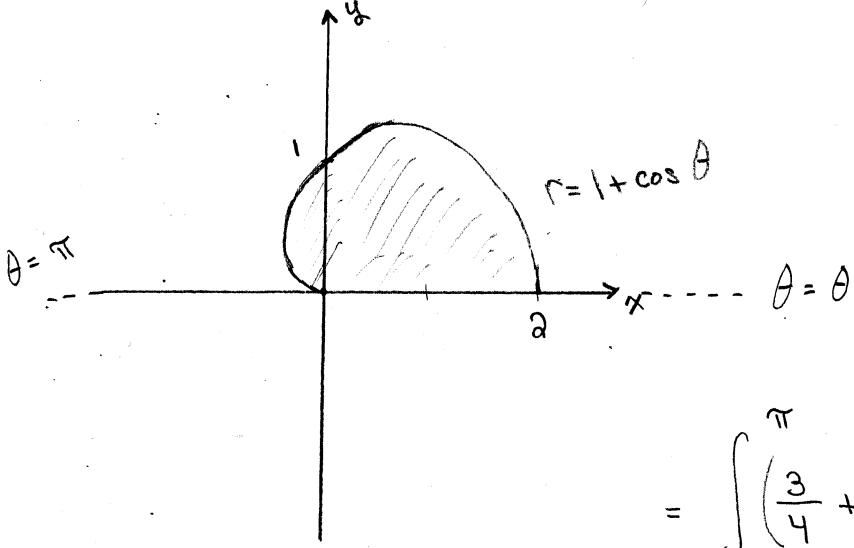
$$\int_{-1}^0 \int_{-\sqrt{y+1}}^{\sqrt{y+1}} y^2 dx dy$$

$$\int_{x=-1}^{x=1} \int_{y=x^2-1}^{y=0} y^2 dy dx$$

$$= \int_{-1}^1 \frac{1}{3} y^3 \Big|_{x^2-1}^0 dx = \int_{-1}^1 -\frac{1}{3} (x^2 - 1)^3 dx$$

$$= 2 \left( -\frac{1}{3} \right) \int_0^1 (x^6 - 3x^4 + 3x^2 - 1) dx = -\frac{2}{3} \left( \frac{1}{7} - \frac{3}{5} + 1 - 1 \right) = \boxed{\frac{32}{105} \approx 0.30476}$$

2. (2 points) Find the area of the upper half of the cardioid  $r = 1 + \cos \theta$ .



$$\int_0^\pi \int_{r=0}^{r=1+\cos\theta} r dr d\theta$$

$$= \int_0^\pi \frac{1}{2} (1 + \cos \theta)^2 d\theta$$

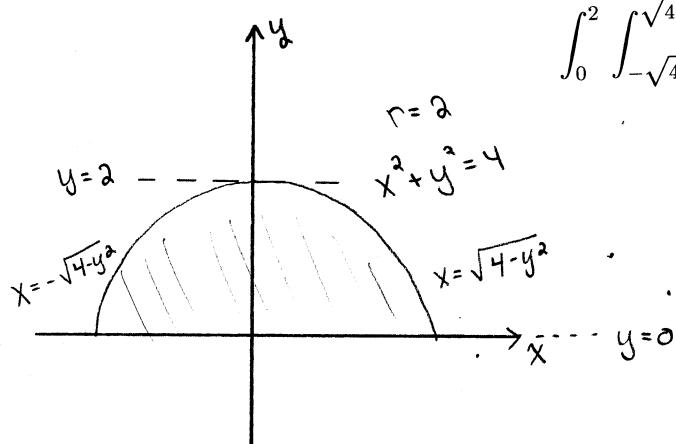
$$= \frac{1}{2} \int_0^\pi 1 + 2\cos \theta + \frac{1}{2}(1 + \cos 2\theta) d\theta$$

$$= \int_0^\pi \left( \frac{3}{4} + \cos \theta + \frac{1}{4} \cos 2\theta \right) d\theta$$

Turn over.

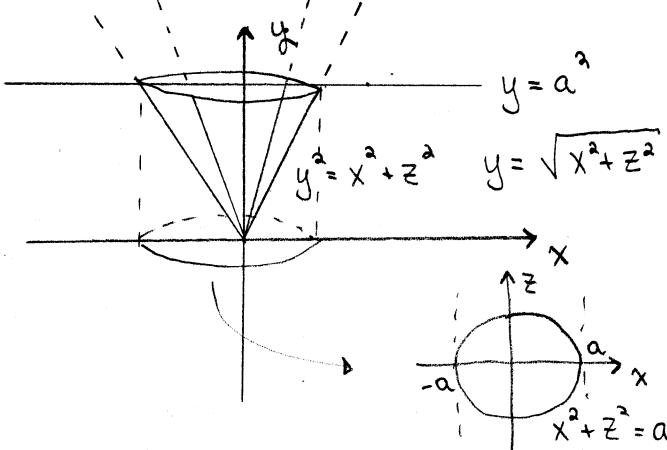
$$= \left. \frac{3}{4} \theta + \sin \theta + \frac{1}{8} \sin 2\theta \right|_0^\pi = \boxed{\frac{3}{4} \pi}$$

3. (2 points) Convert to polar coordinates and evaluate.



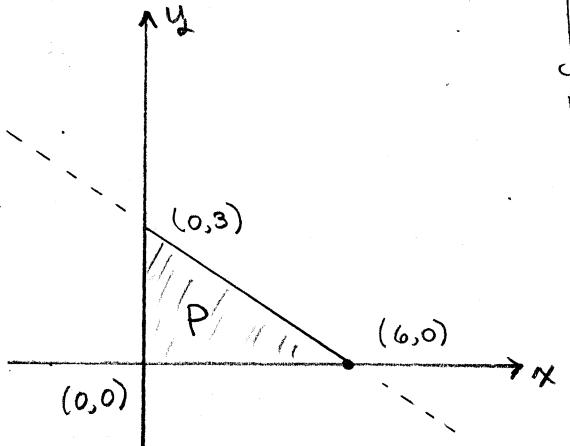
$$\begin{aligned}
 & \int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} (x^2 + y^2)^2 dx dy \\
 &= \int_0^\pi \int_0^2 r^4 r dr d\theta \\
 &= \pi \int_0^2 r^5 dr \\
 &= \pi \left( \frac{1}{6} r^6 \right) \Big|_0^2 = \frac{64\pi}{6} \\
 &= \boxed{\frac{32\pi}{3}}
 \end{aligned}$$

4. (2 points) Set up the triple integral that gives the volume of the solid region  $E$  above the upper half of the cone  $y^2 = x^2 + z^2$  and under the plane  $y = a^2$ , where  $a > 0$ .



$$\begin{aligned}
 & \int_{-a}^a \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \int_{\sqrt{x^2+z^2}}^{a^2} 1 dy dz dx \\
 & \text{or} \\
 & \int_0^{2\pi} \int_0^a \int_{y=r}^{y=a^2} r dy dr d\theta
 \end{aligned}$$

5. (2 points) Find the mass of the thin triangular plate lying in the  $xy$ -plane with vertices  $(0, 0)$ ,  $(0, 3)$ , and  $(6, 0)$  if the density at  $(x, y)$  is given by  $\rho(x, y) = xy$ .



$$\begin{aligned}
 \iint_P xy \, dA &= \int_{y=0}^3 \int_{x=0}^{6-2y} (xy) \, dx \, dy \\
 &= \int_0^3 \frac{1}{2} (6-2y)^2 y \, dy = \int_0^3 (18y - 12y^2 + 2y^3) \, dy \\
 &= 9y^2 - 4y^3 + \frac{1}{2}y^4 \Big|_0^3 \\
 &= 81 - 108 + \frac{81}{2} = \boxed{\frac{27}{2}}
 \end{aligned}$$