Math 233 - Quiz 1

January 21, 2021

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due January 28.

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1. (4 points) Let $\vec{u}$ be the vector from $P(-2, 5)$ to $Q(3, -1)$.

   (a) Find the component form of $\vec{u}$.

$$\vec{u} = \overrightarrow{PQ} = \langle 3-(-2), -1-5 \rangle = \langle 5, -6 \rangle = 5\hat{i} - 6\hat{j}$$

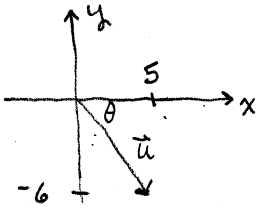
   (b) Compute $\|\vec{u}\|$.

$$\|\vec{u}\| = \sqrt{(5)^2 + (-6)^2} = \sqrt{25+36} = \sqrt{61}$$

   (c) Determine a vector of magnitude 5 whose direction is opposite that of $\vec{u}$.

$$-5\frac{\vec{u}}{\|\vec{u}\|} = -\frac{5}{\sqrt{61}}\vec{u} = -\frac{5}{\sqrt{61}}\left(5\hat{i} - 6\hat{j}\right)$$

   (d) What angle does $\vec{u}$ make with the positive x-axis? Write your answer in degrees, rounded to the nearest tenth.



$$\tan\theta = \frac{-6}{5} \Rightarrow \theta \approx -50.2^\circ$$

OR WE COULD SAY 309.8°

2. (2 points) Let $\vec{u} = -3\hat{i} + 5\hat{j}$ and let $\vec{v}$ be the 2D vector of magnitude 4 that makes a 120° angle with the positive x-axis. Compute $3\vec{u} + \vec{v}$ and then find its magnitude.

$$\vec{u} = -3\hat{i} + 5\hat{j}$$

$$\vec{v} = 4\cos 120^\circ \hat{i} + 4\sin 120^\circ \hat{j}$$

$$= -2\hat{i} + 2\sqrt{3}\hat{j}$$

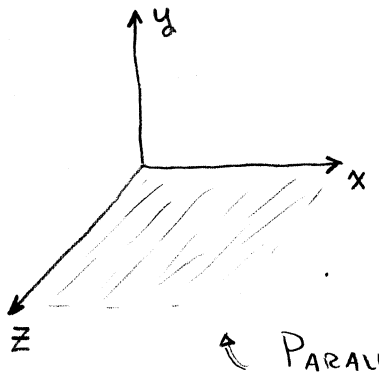
$$3\vec{u} + \vec{v} = (-9\hat{i} + 15\hat{j}) + (-2\hat{i} + 2\sqrt{3}\hat{j})$$

$$3\vec{u} + \vec{v} = -11\hat{i} + \left(15 + 2\sqrt{3}\right)\hat{j}$$

$$\|3\vec{u} + \vec{v}\| = \sqrt{(-11)^2 + \left(15+2\sqrt{3}\right)^2} \approx 21.49$$

Turn over.

3. (1 point) Write an equation of the plane passing through $(1, -3, 2)$ that is parallel to the xz-plane.



$y = -3$

↑ PARALLEL TO THIS PLANE ($y=0$), BUT 3 UNITS DOWN

4. (1 point) Use vectors to show that the points $A(1, 0, 1)$, $B(0, 1, 1)$, and $C(1, 1, 1)$ are not collinear.

$$\overrightarrow{AB} = (0-1)\hat{i} + (1-0)\hat{j} + (1-1)\hat{k} = -\hat{i} + \hat{j}$$

$$\overrightarrow{AC} = (1-1)\hat{i} + (1-0)\hat{j} + (1-1)\hat{k} = \hat{j}$$

$\overrightarrow{AB} \neq t\,\overrightarrow{AC}$

$\overrightarrow{AB}$ AND $\overrightarrow{AC}$ ARE NOT SCALAR MULTIPLES OF EACH OTHER. $\overrightarrow{AC}$ IS NOT PARALLEL TO $\overrightarrow{AB}$. THEY CAN'T BE COLLINEAR.

5. (2 points) Let $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and let $\vec{b} = \vec{v} - \vec{w}$, where $\vec{v} = 2\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{w} = 6\hat{i} + \hat{j} + 2\hat{k}$. Find the measure of the angle between $\vec{a}$ and $\vec{b}$. Write your answer in radians, rounded to the nearest hundredth.

$$\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{b} = \vec{v} - \vec{w} = (2\hat{i} + \hat{j} + 4\hat{k}) - (6\hat{i} + \hat{j} + 2\hat{k})$$

$$= -4\hat{i} + 2\hat{k}$$

$\theta \approx 2.07$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|\,\|\vec{b}\|} = \frac{-12 + 0 + 4}{\sqrt{14}\,\sqrt{20}} = \frac{-8}{\sqrt{280}}$$