

# Math 233 - Quiz 2

January 28, 2021

Name \_\_\_\_\_

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary. This quiz is due February 4.

1. (1.5 points) Find a vector of magnitude 3 that is orthogonal to both  $\vec{u} = 3\hat{i} - 2\hat{j} + \hat{k}$  and  $\vec{w} = 5\hat{i} + 2\hat{j} + 2\hat{k}$ .

$$\vec{u} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 5 & 2 & 2 \end{vmatrix}$$

$$\begin{aligned} &= \hat{i}(-6) - \hat{j}(1) + \hat{k}(16) \\ &= -6\hat{i} - \hat{j} + 16\hat{k} \end{aligned}$$

$$\|\vec{u} \times \vec{w}\| = \sqrt{36 + 1 + 256} = \sqrt{293}$$

Answer:

$$\frac{3}{\sqrt{293}} (\vec{u} \times \vec{w}) = \frac{-18\hat{i} - 3\hat{j} + 48\hat{k}}{\sqrt{293}}$$

2. (2 points) Use vectors to determine the area of the triangle with vertices at  $A(0, 1, 3)$ ,  $B(-3, 2, 1)$  and  $C(2, 3, 2)$ .

$$\vec{AB} = -3\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{AC} = 2\hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & -2 \\ 2 & 2 & -1 \end{vmatrix}$$

$$= \hat{i}(3) - \hat{j}(7) + \hat{k}(-8)$$

$$\vec{AB} \times \vec{AC} = 3\hat{i} - 7\hat{j} - 8\hat{k}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \|\vec{AB} \times \vec{AC}\|$$

$$= \frac{1}{2} \sqrt{9 + 49 + 64} = \frac{1}{2} \sqrt{122}$$

$\approx 5.523$

3. (1.5 points) Find the volume of the parallelepiped determined by the vectors  $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ ,  $\vec{b} = -\hat{i} + \hat{j} - 4\hat{k}$ , and  $\vec{c} = 4\hat{i} + 2\hat{j} - \hat{k}$ .

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 3 & -2 & 1 \\ -1 & 1 & -4 \\ 4 & 2 & -1 \end{vmatrix}$$

$$= (3)(7) - (-2)(17) + (1)(-6)$$

$$= 21 + 34 - 6 = 49$$

$$\text{Volume} = |\vec{a} \cdot (\vec{b} \times \vec{c})|$$

$$= 49$$

Turn over.

4. (2 points) Find parametric and symmetric equations for the line in space passing through the points  $P(2, 3, -1)$  and  $Q(3, 1, 1)$ .

$$\vec{r} = \vec{PQ} = \hat{i} - 2\hat{j} + 2\hat{k}$$

Using  $P(2, 3, -1)$

PARAMETRIC

$$\begin{aligned}x &= 2 + t \\y &= 3 - 2t \\z &= -1 + 2t\end{aligned}$$

SYMMETRIC

$$x - 2 = \frac{y - 3}{-2} = \frac{z + 1}{2}$$

5. (1 point) Refer back to problem 2. Find an equation for the plane containing the triangle,  $\triangle ABC$ .

$$\vec{n} = \vec{AB} \times \vec{AC} = 3\hat{i} - 7\hat{j} - 8\hat{k}$$

$$3x - 7y - 8z = -31$$

Using  $A(0, 1, 3)$  ...

$$3(x-0) + (-7)(y-1) + (-8)(z-3) = 0$$

6. (2 points) Consider the plane described by the equation  $2x - y + 3z = 7$ .

- (a) Find a unit vector perpendicular to the plane.

$$\vec{n} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$\begin{aligned}\|\vec{n}\| &= \sqrt{4 + 1 + 9} \\&= \sqrt{14}\end{aligned}$$

$$\frac{\vec{n}}{\|\vec{n}\|} = \frac{2\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{14}}$$

- (b) Find the distance from the plane to the point  $P(1, 1, 1)$ .

Using Formula  $D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$  ...

Derivation is  
in Textbook

$$D = \frac{|2(1) - (1) + 3(1) - 7|}{\sqrt{4 + 1 + 9}} = \frac{3}{\sqrt{14}}$$