

Math 233 - Quiz 5

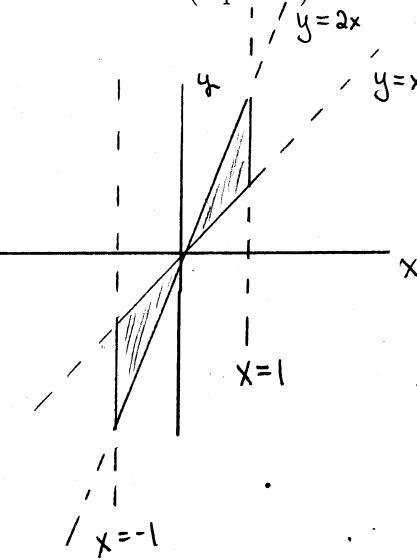
April 8, 2021

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due April 15.

1. (3 points) Evaluate the iterated integral and sketch the region of integration.



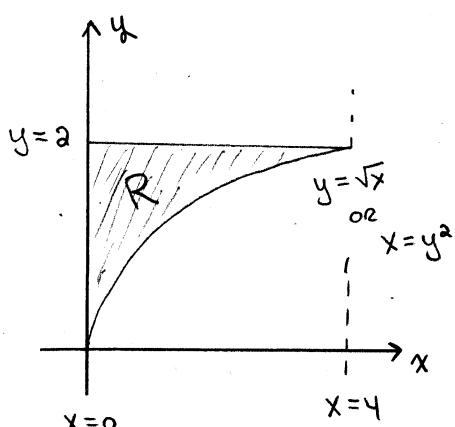
$$\begin{aligned} & \int_{-1}^1 \int_x^{2x} e^{x+y} dy dx \\ &= \int_{-1}^1 e^{x+y} \Big|_{y=x}^{y=2x} dx \\ &= \int_{-1}^1 (e^{3x} - e^{2x}) dx \end{aligned}$$

* THIS IS REALLY NOT A
GOOD REGION
SINCE THE "TOP" AND
"BOTTOM" CURVES
ARE BACKWARD ON
THE LEFT OF
 $x=0$.

$$= \left. \frac{1}{3} e^{3x} - \frac{1}{2} e^{2x} \right|_{-1}^1 = \left\{ \frac{1}{3} e^3 - \frac{1}{2} e^2 \right. \\ \left. - \frac{1}{3} e^{-3} + \frac{1}{2} e^{-2} \right\}$$

≈ 3.05

2. (3 points) Evaluate the iterated integral by reversing the order of integration.



$$\int_0^4 \int_{\sqrt{x}}^2 \sin y^3 dy dx$$

$$\int_{y=0}^2 \int_{x=y^2}^{x=4} \sin y^3 dx dy$$

$$= \int_0^2 x \sin y^3 \Big|_{y=0}^{y=\sqrt{x}} dy = \int_0^2 y^2 \sin y^3 dy$$

$u = y^3$
 $du = 3y^2 dy$
Turn over.

$$= \frac{1}{3} \int_0^8 \sin u du = -\frac{1}{3} \cos u \Big|_0^8 = \left\{ -\frac{1}{3} \cos 8 + \frac{1}{3} \right. \\ \left. \approx 0.3818 \right\}$$

3. (3 points) Evaluate by converting to polar coordinates.

$$\int_0^1 \int_x^{\sqrt{2-x^2}} (x+2y) dy dx$$

$$= \int_{\pi/4}^{\pi/2} \int_0^{\sqrt{2}} (r \cos \theta + 2r \sin \theta) r dr d\theta$$

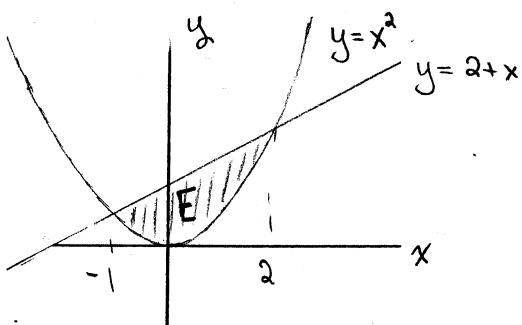
$$= \int_{\pi/4}^{\pi/2} (\cos \theta + 2 \sin \theta) d\theta \cdot \int_0^{\sqrt{2}} r^2 dr$$

$$= \left(\sin \theta - 2 \cos \theta \Big|_{\pi/4}^{\pi/2} \right) \left(\frac{1}{3} r^3 \Big|_0^{\sqrt{2}} \right)$$

$$= \left(1 - \frac{\sqrt{2}}{2} + \sqrt{2} \right) \left(\frac{\sqrt{8}}{3} \right) = \boxed{\frac{\sqrt{8}}{3} + \frac{2}{3} \approx 1.6095}$$

4. (1 points) Let E be the plane region between the graphs of $y = x^2$ and $y = x + 2$. Sketch the region E and write the iterated integral (in the $dy dx$ order) for the double integral given below. Do not evaluate.

$$\iint_E (xy + 5) dA,$$



$$\int_{x=-1}^{x=2} \int_{y=x^2}^{y=x+2} (xy + 5) dy dx$$

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$