

Math 233 - Test 1

February 11, 2021

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) Forces with magnitudes 200 pounds and 100 pounds act on a hitch at angles of 60° and -30° , respectively, with the x -axis. Find the direction and magnitude of the resultant force.

$$\vec{f}_1 = 200 \cos 60^\circ \hat{i} + 200 \sin 60^\circ \hat{j} = 100\hat{i} + 100\sqrt{3}\hat{j}$$

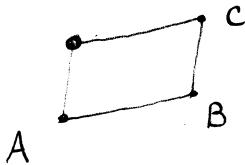
$$\vec{f}_2 = 100 \cos(-30^\circ) \hat{i} + 100 \sin(-30^\circ) \hat{j} = 50\sqrt{3}\hat{i} - 50\hat{j}$$

$$\boxed{\vec{f}_1 + \vec{f}_2 = (100 + 50\sqrt{3})\hat{i} + (100\sqrt{3} - 50)\hat{j}}$$

$$\tan^{-1} \left(\frac{100\sqrt{3} - 50}{100 + 50\sqrt{3}} \right) \approx 33.435^\circ$$

$$\|\vec{f}_1 + \vec{f}_2\| = \sqrt{(100 + 50\sqrt{3})^2 + (100\sqrt{3} - 50)^2} \approx \boxed{223.609 \text{ lb}}$$

2. (4 points) Determine the coordinates of point D such that $ABCD$ is a parallelogram with $A(1, 1)$, $B(2, 4)$, and $C(7, 4)$.



Go \vec{BC} from A ...

$$\vec{BC} = \langle 5, 0 \rangle$$

STARTING AT A AND MOVING MAG. AND DIRECTION
OF \vec{BC} PUTS US AT $\boxed{D(6, 1)}$

3. (4 points) The point $M(1, -7, 4)$ is the midpoint of the line segment PQ , where $P(-2, 5, -8)$. Determine the vector \vec{PQ} .

$$\vec{PQ} = 2\vec{PM} = 2 \langle 1 - (-2), -7 - 5, 4 - (-8) \rangle$$

$$= 2 \langle 3, -12, 12 \rangle$$

$$= \boxed{6\hat{i} - 24\hat{j} + 24\hat{k}}$$

4. (4 points) Determine the vector of magnitude 8 that is in the opposite direction of $\vec{w} = 3\hat{i} - 5\hat{j} + 7\hat{k}$.

$$\frac{-8\vec{\omega}}{\|\vec{\omega}\|} = \frac{-24\hat{i} + 40\hat{j} - 56\hat{k}}{\sqrt{9+25+49}}$$

$$= \frac{-24\hat{i} + 40\hat{j} - 56\hat{k}}{\sqrt{83}}$$

5. (6 points) Show that the following points are collinear: $A(-5, 1, 3)$, $B(10, 6, 13)$, and $C(-11, -1, -1)$.

$$\vec{AB} = 15\hat{i} + 5\hat{j} + 10\hat{k}$$

$$\vec{AC} = -6\hat{i} - 2\hat{j} - 4\hat{k}$$

$$\vec{AB} = -\frac{5}{3}\vec{AC} \Rightarrow \vec{AB} \text{ & } \vec{AC} \text{ ARE PARALLEL}$$

AND SHARE POINT A.

6. (5 points) Let γ be the angle that $\vec{u} = \hat{i} + 3\hat{j} - 2\hat{k}$ makes with the positive z -axis. Find the measure of γ . Give your final answer in degrees, rounded to the nearest hundredth.

Angle between \vec{u} & \hat{k} ...

$$\cos \gamma = \frac{\vec{u} \cdot \hat{k}}{\|\vec{u}\| \|\hat{k}\|} = \frac{-2}{\sqrt{1+9+4}} = \frac{-2}{\sqrt{14}}$$

$$\Rightarrow \gamma \approx 122.31^\circ$$

7. (5 points) Let $\vec{v} = 2\hat{i} + 3\hat{j}$ and $\vec{w} = -3\hat{i} - \hat{j} - 5\hat{k}$. Compute $\text{proj}_{\vec{v}} \vec{w}$.

$$\text{proj}_{\vec{v}} \vec{w} = \frac{\vec{w} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v}$$

$$= \frac{-6 - 3 + 0}{4 + 9} \vec{v} = -\frac{9}{13} \vec{v} = -\frac{18}{13} \hat{i} - \frac{27}{13} \hat{j}$$

8. (6 points) Find all real numbers t for which $\vec{u} = 2t\hat{i} + t\hat{j} - 3\hat{k}$ is orthogonal to $\vec{v} = (t+1)\hat{i} + 6\hat{j} + (t+1)\hat{k}$.

$$\vec{u} \cdot \vec{v} = 0$$

$$\Rightarrow 2t(t+1) + 6t - 3(t+1) = 0$$

$$\Rightarrow 2t^2 + 5t - 3 = 0$$

$$\Rightarrow (2t - 1)(t + 3) = 0$$

$$t = \frac{1}{2}, t = -3$$

9. (6 points) Find a set of symmetric equations for the line passing through the points $P(1, 2, 3)$ and $Q(-4, -1, 5)$.

$$\vec{PQ} = -5\hat{i} - 3\hat{j} + 2\hat{k}$$

Using $P(1, 2, 3) \dots$

$$\frac{x-1}{-5} = \frac{y-2}{-3} = \frac{z-3}{2}$$

10. (6 points) The points $A(2, 0, 3)$, $B(5, -1, -2)$, $C(3, 2, 5)$, and $D(0, 3, 10)$ are the vertices of parallelogram $ABCD$. Find the area of the parallelogram.

$$\text{Area} = \|\vec{AB} \times \vec{AD}\|$$

$$\vec{AB} \times \vec{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 5 \\ -2 & 3 & 7 \end{vmatrix}$$

$$= \hat{i}(8) - \hat{j}(11) + \hat{k}(7)$$

$$\vec{AB} = 3\hat{i} - \hat{j} - 5\hat{k}$$

$$\vec{AD} = -2\hat{i} + 3\hat{j} + 7\hat{k}$$

$$\text{Area} = \sqrt{64 + 121 + 49} = \sqrt{234}$$

$$= 3\sqrt{26}$$

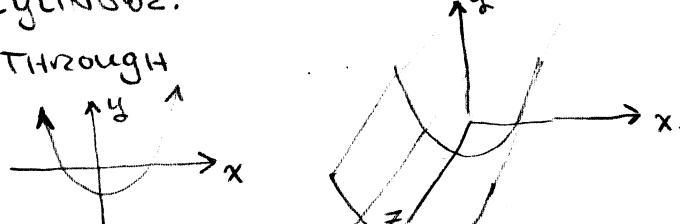
≈ 15.30

11. (5 points) Sketch or describe the 3D surface defined by the equation $y = x^2 - 1$.

THE SURFACE IS A PARABOLIC CYLINDER.

IT COMES OUT OF THE PAPER, THROUGH

THE 2D CURVE $y = x^2 - 1$.

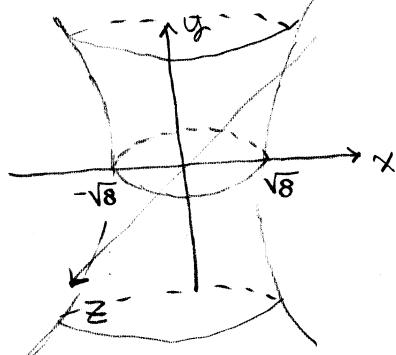


12. (10 points) Identify the quadric surface. Choose either one of the two and draw a rough sketch of the graph.

(a) $2x^2 - y^2 + 2z^2 = 8$

HYPERBOLOID OF ONE SHEET

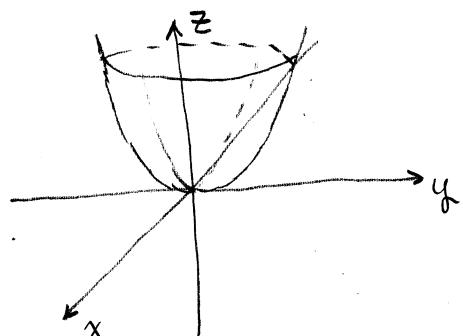
(CROSS SECTIONS UP Y-AXIS
ARE CIRCLES.)



(b) $x^2 + 4y^2 - z = 0$

ELLiptic PARABOLOID

(CROSS SECTIONS UP Z-AXIS
ARE ELLIPSES.)



13. (14 points) Consider the planes described by the following equations:

$$x - 2y + z = 6 \quad \text{and} \quad -2x + y + 5z = -3.$$

(a) Show that the planes are not parallel.

$$\vec{n}_1 = \hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{n}_2 = -2\hat{i} + \hat{j} + 5\hat{k}$$

\vec{n}_1 AND \vec{n}_2 ARE NOT SCALAR MULTIPLES.

NORMAL VECTORS ARE NOT PARALLEL.

(b) Find the measure of the acute angle between the planes. Write your answer in degrees, rounded to the nearest hundredth.

$$\cos \theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \|\vec{n}_2\|} = \frac{-2 - 2 + 5}{\sqrt{6} \sqrt{30}} = \frac{1}{\sqrt{180}}$$

$$\theta \approx 85.73^\circ$$

DIRECTION... (c) Find a set of parametric equations for the line of intersection.

$$\begin{aligned}\vec{n}_1 \times \vec{n}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ -2 & 1 & 5 \end{vmatrix} \\ &= \hat{i}(-11) - \hat{j}(7) + \hat{k}(-3) \\ &= -11\hat{i} - 7\hat{j} - 3\hat{k}\end{aligned}$$

$$\begin{aligned}\text{POINT...} \\ x = 0 \\ -2y + z = 6 \\ y + 5z = -3 \\ \hline z = 0 \\ y = -3 \\ \text{LINE...}\end{aligned}$$

$$(0, -3, 0)$$

$$\begin{aligned}x &= -11t \\ y &= -3 - 7t \\ z &= -3t\end{aligned}$$

14. (8 points) Show that the lines are perpendicular.

$$L_1 : \quad x = 9 + 6t, \quad y = 2 + 2t, \quad z = -3 - 2t$$

$$L_2 : \quad \frac{x-3}{2} = -\frac{y}{5} = z+1$$

L_1 DIRECTION:

$$\vec{u} = 6\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\left. \begin{array}{l} \\ \end{array} \right\} \vec{u} \cdot \vec{v} = 12 - 10 - 2 \\ = 0$$

L_2 DIRECTION:

$$\vec{v} = 2\hat{i} - 5\hat{j} + \hat{k}$$

VECTORS ARE
ORTHOG. ✓

Common pt ?

$$\frac{9 + 6t - 3}{2} = \frac{2 + 2t}{-5} \\ = -3 - 2t + 1$$

$t = -1$ MAKES
THEM EQUAL.

INTERSECTION PT

$$\text{is } (3, 0, -1)$$

✓

15. (9 points) Find an equation of the plane passing through the three points $P(1, 1, 3)$, $Q(3, -2, 4)$ and $R(0, 1, -2)$.

$$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ -1 & 0 & -5 \end{vmatrix} = \hat{i}(15) - \hat{j}(-9) + \hat{k}(-3) \\ = 15\hat{i} + 9\hat{j} - 3\hat{k}$$

Using $P(1, 1, 3)$...

$$15(x-1) + 9(y-1) - 3(z-3) = 0$$

OR

$$15x + 9y - 3z - 15 = 0$$