Math 233 - Test 1

February 11, 2021

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) Forces with magnitudes 200 pounds and 100 pounds act on a hitch at angles of $60°$ and $-30°$, respectively, with the x-axis. Find the direction and magnitude of the resultant force.

2. (4 points) Determine the coordinates of point D such that $ABCD$ is a parallelogram with $A\,(1,1)$, $B\,(2,4)$, and $C\,(7,4)$.

3. (4 points) The point $M\,(1,-7,4)$ is the midpoint of the line segment PQ, where $P\,(-2,5,-8)$. Determine the vector $\vec{PQ}$.

4. (4 points) Determine the vector of magnitude 8 that is in the opposite direction of $\vec{w} = 3\hat{\imath} - 5\hat{\jmath} + 7\hat{k}$.

5. (6 points) Show that the following points are collinear: $A\,(-5, 1, 3)$, $B\,(10, 6, 13)$, and $C\,(-11, -1, -1)$.

6. (5 points) Let γ be the angle that $\vec{u} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ makes with the positive z-axis. Find the measure of γ. Give your final answer in degrees, rounded to the nearest hundredth.

7. (5 points) Let $\vec{v} = 2\hat{\imath} + 3\hat{\jmath}$ and $\vec{w} = -3\hat{\imath} - \hat{\jmath} - 5\hat{k}$. Compute $\text{proj}_{\vec{v}}\,\vec{w}$.

8. (6 points) Find all real numbers t for which $\vec{u} = 2t\hat{\imath} + t\hat{\jmath} - 3\hat{k}$ is orthogonal to $\vec{v} = (t+1)\hat{\imath} + 6\hat{\jmath} + (t+1)\hat{k}$.

9. (6 points) Find a set of symmetric equations for the line passing through the points $P\,(1, 2, 3)$ and $Q\,(-4, -1, 5)$.

10. (6 points) The points $A\,(2,0,3)$, $B\,(5,-1,-2)$, $C\,(3,2,5)$, and $D\,(0,3,10)$ are the vertices of parallelogram $ABCD$. Find the area of the parallelogram.

11. (5 points) Sketch or describe the 3D surface defined by the equation $y = x^2 - 1$.

12. (10 points) Identify the quadric surface. Choose either one of the two and draw a rough sketch of the graph.

    (a) $2x^2 - y^2 + 2z^2 = 8$

    (b) $x^2 + 4y^2 - z = 0$

13. (14 points) Consider the planes described by the following equations:

$$x - 2y + z = 6 \qquad \text{and} \qquad -2x + y + 5z = -3.$$

(a) Show that the planes are not parallel.

(b) Find the measure of the acute angle between the planes. Write your answer in degrees, rounded to the nearest hundredth.

(c) Find a set of parametric equations for the line of intersection.

14. (8 points) Show that the lines are perpendicular.

$$L_1: \qquad x = 9 + 6t, \quad y = 2 + 2t, \quad z = -3 - 2t$$

$$L_2: \qquad \frac{x-3}{2} = -\frac{y}{5} = z + 1$$

15. (9 points) Find an equation of the plane passing through the three points $P\,(1, 1, 3)$, $Q\,(3, -2, 4)$ and $R\,(0, 1, -2)$.