Math 233 - Test 2

March 11, 2021

Name ________________________

Score __________

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) Consider the vector-valued function defined by $\vec{r}(t) = t^3\hat{\imath} + 2t\hat{\jmath}$.

   (a) Sketch the graph of $\vec{r}(t)$. Use arrows to show the orientation of the curve. (You may wish to do part (b) first.)

   (b) Eliminate the parameter t to find an equation for the graph of $\vec{r}$ in the rectangular coordinates x and y.

2. (3 points) What is the domain of $\vec{r}(t) = \dfrac{1}{2t+1}\,\hat{\imath} + \sqrt{t-3}\,\hat{\jmath} + (\ln t)\,\hat{k}$?

3. (5 points) Find a vector-valued function whose graph is the line in space passing through the points $P\,(1, 2, -1)$ and $Q\,(2, 7, 5)$.

4. (5 points) Let $\vec{r}(t) = \sqrt[3]{t}\,\hat{\imath} + \dfrac{1}{t+1}\,\hat{\jmath} + e^{-t}\,\hat{k}$. Evaluate the definite integral $\displaystyle\int_0^1 \vec{r}(t)\,dt$.

5. (4 points) Let $\vec{r}(t) = 5\cos(\pi t)\,\hat{\imath} - 5\sin(\pi t)\,\hat{\jmath}$. Show that $\vec{v}(t)$ is always orthogonal to $\vec{r}(t)$.

6. (8 points) At $t = 0$, an object starts from rest at the point $P\,(1, 2, 0)$ and moves with acceleration $\vec{a}(t) = \hat{\jmath} + 2\hat{k}$. Find the location of the object at $t = 2$.

7. (10 points) Let $\vec{r}(t) = 2\sin(t)\,\hat{\imath} + 5t\,\hat{\jmath} + 2\cos(t)\,\hat{k}$.

   (a) Describe the graph of $\vec{r}(t)$.

   (b) Find the principal unit normal vector for $\vec{r}(t)$.

8. (6 points) Parameterize the line described by $\vec{r}(t) = (3-3t)\,\hat{\imath} + 4t\,\hat{\jmath}$ using the arc-length parameter s, from $t = 0$.

9. (8 points) A golf ball is hit in a horizontal direction with speed v_0 off the top edge of a building that is 100 ft tall.

   (a) Using $g = 32\,\text{ft/s}^2$ and ignoring air resistance, write the vector-valued function that gives the position of the golf ball at time t.

   (b) Find v_0 if the ball lands 450 ft away from the building.

10. (3 points) Let $w(x, y, z) = x^2 + y^2 + z^2$. Describe, in detail, the level surface $w(x, y, z) = 9$.

11. (10 points) Consider the function defined by the equation $z = x^2 + y^2$.

(a) What is the domain of this function?

(b) What is the range of this function?

(c) Sketch the level curve $z = 1$.

(d) Sketch the level curve $y = 1$.

(e) Describe (or sketch) the graph of $z = x^2 + y^2$.

12. (8 points) Find k so that f is continuous everywhere.

$$f(x, y) = \begin{cases} \dfrac{x^4 - 4y^2}{x^2 + 2y^2}, & (x, y) \neq (0, 0) \\ k, & (x, y) = (0, 0) \end{cases}$$

13. (8 points) Consider the following limit: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy+y^3}{x^2+y^2}$.

(a) Show that the limit does not exist.

(b) Now think about this new limit: $\displaystyle\lim_{(x,y)\to(1,0)} \frac{(x-1)y+y^3}{(x-1)^2+y^2}$.
This limit also does not exist. What paths could you use to prove it?

14. (5 points) Let $z = \ln(xy+y^2)$. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

15. (5 points) Let $f(x, y, z) = xyz$.

(a) Before computing them, what do you expect to be true of f_{xyy}, f_{yxy}, and f_{yyx}? Why?

(b) Compute each of the mixed partials mentioned in part (a).

16. (4 points) Suppose you are given the function $f(x, y)$. Further suppose that $f_x(2, 1)$ and $f_y(2, 1)$ both exist. Explain what the number $f_x(2, 1)$ actually measures. How is that different from what the number $f_y(2, 1)$ measures?