Math 233 - Test 3a

April 15, 2021

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (5 points) Let $f(x, y) = x^2 + 5xy - y^2$. Use differentials to approximate the change in $f(x, y)$ as x changes from 2.00 to 1.97 and y changes from 3.00 to 3.05.

2. (5 points) Assume that y is implicitly defined as a function of x by the equation $xe^y + ye^x = 2x^2y$. Use partial derivatives to find dy/dx.

3. (8 points) Find the directional derivative of $f(x, y) = \ln(x^2 + y^2)$ at the point $P\,(1, 2)$ in the direction of $\vec{v} = 3\hat{\imath} - 4\hat{\jmath}$.

Follow-up: If you were standing at the point P on the the graph of f looking in the direction of $\vec{v}$, would you be looking downhill or uphill? How do you know?

4. (12 points) Find and classify the critical points of $g(x, y) = x^2 + x - 3xy + y^3 - 5$.

5. (5 points) The electric voltage in a certain region in space is described by the function $V(x, y, z) = 5x^2 - 3xy + xyz$. At the point $(3, 4, 5)$, in what direction is the voltage increasing most rapidly? Give your answer as a unit vector.

6. (5 points) Suppose w is a function of x, y, z and x, y, z are functions of s, t. Write the chain rule formulas for $\dfrac{\partial w}{\partial s}$ and $\dfrac{\partial w}{\partial t}$.