Math 233 - Test 3b

Name _____

April 15, 2021

Score _____

Show all work to receive full credit. Supply explanations where necessary. This test is due April 22.

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1. (6 points) Find a set of parametric equations for the line normal to the graph of $x^2 - 8xyz + y^2 + 6z^2 = 0$ at the point $P\,(1, 1, 1)$.

2. (6 points) Let $f(x, y) = \sqrt{20 - x^2 - 7y^2}$. Find the linearization of f at $(2, 1)$ and use it to approximate $f(1.98, 1.04)$. (The textbook uses the name *linear approximation* instead of linearization.)

3. (8 points) Use the definition of differentiability to show that $g(x, y) = x^3 + 2xy - y$ is differentiable everywhere in $\mathbb{R}^2$.

4. (5 points) Find a vector that is normal to the surface $z = x^2 - 2xy + 5y^2 + 3x - 2y + 4$ at the point $(1, 1, 9)$.

5. (15 points) Consider the iterated integral $\int_0^1 \int_{x-1}^{1-x} x \, dy \, dx$.

(a) Sketch the region of integration.

(b) Evaluate the iterated integral in the given order.

(c) Reverse the order of integration and write the new equivalent iterated integral(s). DO NOT EVALUATE.

6. (10 points) Find the volume of the solid region under the surface $z = 2x + y^2$ and above the bounded region in the xy-plane between the graphs of $y = x^2$ and $y = x$.

7. (10 points) Consider the double integral

$$\iint_D \frac{3}{1+y^2}\, dA,$$

where D is the region in the xy-plane bounded by the graphs of $x = 0$, $y = \sqrt{x}$, and $y = 2$. Write as an iterated integral and evaluate. (Use whichever order of integration you prefer.)