Math 233 - Final Exam A

Name _____

May 8, 2021

Score _____

Show all work to receive full credit. Each problem is worth 5 points. Place your final answer in the box provided. This test is due May 13.

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1. A baseball, hit 3 feet above the ground, leaves the bat at an angle of 45° and is caught by an outfielder at a height of 3 feet above the ground and 300 feet from home plate. What is the initial speed of the ball? (To receive full credit, you must write and use the vector-valued function $\vec{r}(t)$ that gives the position of the ball at time t. Also ignore air resistance and use $g \approx 32\,\mathrm{ft\,s^{-2}}$.)

2. Find the limit or show that it does not exist: $\lim\limits_{(x,y)\to(2,1)} \dfrac{x-y-1}{\sqrt{x-y}-1}$

3. Find the limit or show that it does not exist: $\lim_{(x,y)\to(1,1)} \dfrac{x^2 - y^2 - 2x + 2y}{2x^2 - 4x + y + 1}$

4. Find an equation of the plane tangent to the graph of $z = \tan^{-1}(y/x)$ at the point $\left(1, 1, \dfrac{\pi}{4}\right)$.

5. Reverse the order of integration and evaluate.

$$\int_0^{\ln 5} \int_{e^x}^{5} \frac{1}{\ln y}\, dy\, dx$$

6. Evaluate the iterated integral by first converting to cylindrical coordinates.

$$\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{x^2+y^2}^{9} dz\, dy\, dx$$

7. A solid in space is bounded above by the elliptic paraboloid $z = 8 - 2x^2 - y^2$ and below by the cylinder $z = y^2$. The density of the solid at the point (x, y, z) is given by $\rho(x, y, z) = z + x^2 + y^2$. Set up the iterated integral that gives the mass of the solid. DO NOT EVALUATE. (You may use whichever coordinate system you prefer.)

8. Let $\vec{F}(x, y, z) = y\hat{\imath} + x^2\hat{\jmath} - z\hat{k}$. Evaluate $\displaystyle\int_C \vec{F}(x, y, z) \cdot d\vec{r}$, where C is the line segment from $(0, 1, 1)$ to $(1, 3, 5)$.