Math 233 - Final Exam B

Name ______________________________

May 13, 2021

Score ____________

Show all work to receive full credit. Each problem is worth 5 points. Place your final answer in the box provided.

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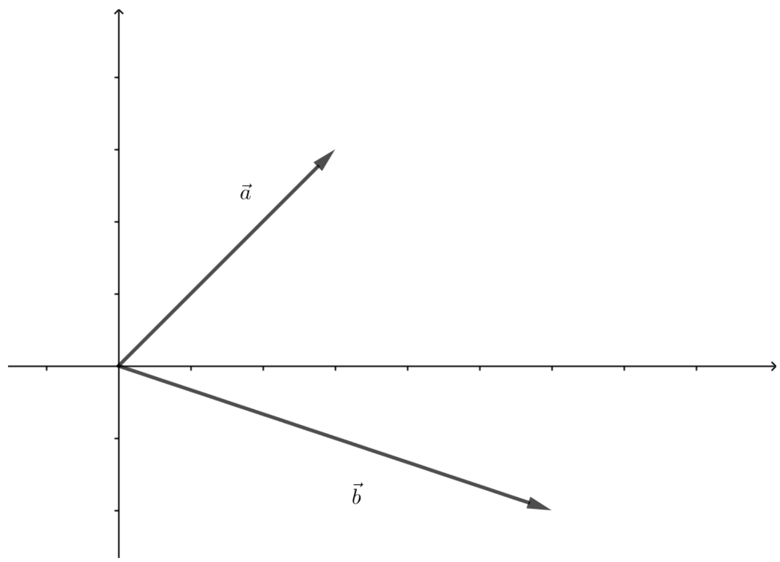
1. The vectors $\vec{a} = 3\hat{\imath} + 3\hat{\jmath}$ and $\vec{b} = 6\hat{\imath} - 2\hat{\jmath}$ are shown below. Sketch and compute $\text{proj}_{\vec{b}}\,\vec{a}$.



2. Given the points $A\,(5,3,1)$, $B\,(3,2,3)$, and $C\,(-4,-1,2)$, find a nonzero vector that is orthogonal to both $\vec{AB}$ and $\vec{AC}$.

3. The line ℓ has the symmetric equations $-x = \dfrac{y-1}{3} = \dfrac{z}{2}$. Find a set of parametric equations for the line through $(5, 2, -4)$ and parallel to ℓ.

4. Find the measure of the acute angle between the planes given below. Give your final answer in degrees, rounded to the nearest tenth.

$$3x + 2y - 7z = 0$$

$$-4x + 4y + 2z = 13$$

5. Find the length of the graph of $\vec{r}(t) = (3t-1)\hat{\imath} + (2t+7)\hat{\jmath} - (t-5)\hat{k}$ from the point where $t = 1$ to the point where $t = 3$.

6. Suppose f is a function of the three variables x, y, and z. Choose two different fourth-order partial derivatives of f and state the conditions under which they will be equal to one another.

7. Use differentials to estimate the change in $f(x, y, z) = x^2 \ln(5yz+1)$ as (x, y, z) changes from $(2, 1, 1)$ to $(1.99, 1.02, 1.01)$. Round your final answer to four decimal places.

8. Find the directional derivative of $g(x, y, z) = xye^z$ at $P\,(2, 4, 0)$ in the direction toward the point $Q\,(0, 0, 0)$.

9. Over a certain region in space the electrical potential V is given by

$$V(x, y, z) = 5x^2 - 3xy + xyz.$$

At the point $(3, 4, 5)$, in which direction does the potential decrease most rapidly?

10. Find the critical point(s) of $f(x, y) = x^2 + xy + 2y^2 + x - 3y + 10$. Then use the 2nd-partials test to classify the critical point(s).

11. Let P be the plane region between the graphs of $y = x^2$ and $y = x + 2$. Sketch the region P and then evaluate the double integral given below.

$$\iint_P (x+2)\, dA$$

12. Evaluate the iterated integral by converting to polar coordinates.

$$\int_0^2 \int_y^{\sqrt{8-y^2}} \sqrt{x^2+y^2}\, dx\, dy$$