

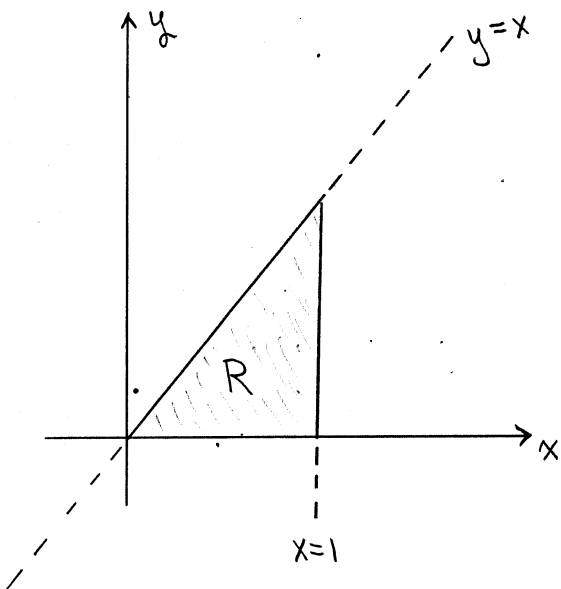
Math 233 - Quiz 10

April 21, 2022

Name key Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Find the volume of the solid whose base is the triangle in the xy -plane bounded by the x -axis, the line $y = x$ and the line $x = 1$, while the top of the solid is the plane $z = 3 - x - y$.



$$V = \iint_R (3-x-y) dA$$
$$= \int_{x=0}^{x=1} \int_{y=0}^{y=x} (3-x-y) dy dx$$

$$= \int_0^1 \left(3y - xy - \frac{1}{2}y^2 \right) \Big|_0^x dx$$

$$= \int_0^1 \left(3x - \frac{3}{2}x^2 \right) dx = \left(\frac{3}{2}x^2 - \frac{3}{6}x^3 \right) \Big|_0^1$$

$$= \frac{3}{2} - \frac{3}{6} = \boxed{1}$$

Turn over.

2. (5 points) Evaluate $\iint_A \frac{\sin x}{x} dA$, where A is the same triangle as in the problem above.

REFERENCING BACK TO THE REGION IN PROB 1...

$$\iint_A \frac{\sin x}{x} dA = \int_{x=0}^{x=1} \int_{y=0}^{y=x} \frac{\sin x}{x} dy dx$$

$$= \int_0^1 \left. \frac{y \sin x}{x} \right|_{y=0}^{y=x} dx = \int_0^1 \sin x dx$$

$$= -\cos x \Big|_0^1 = -\cos(1) + 1$$

$$= \boxed{1 - \cos(1)}$$