Math 233 - Quiz 1

January 20, 2022

Name key

Score _____

This quiz is available in Canvas. It is due January 25.

1. (1 point) Which of these is a unit vector with the direction from $P(1,-3)$ to $Q(-4,9)$?

   (a) $\langle -5/13, 12/13\rangle$
   (b) $\langle 5/13, -12/13\rangle$
   (c) $\langle -5/13, -12/13\rangle$
   (d) $\langle -5, 12\rangle$

$\vec{PQ} = (-4-1)\hat{i} + (9-(-3))\hat{j}$

$= -5\hat{i} + 12\hat{j}$

$\|\vec{PQ}\| = \sqrt{25+144} = \sqrt{169} = 13$

2. (1 point) Determine a vector of magnitude 8 that has the opposite direction of $\vec{w} = \langle 3, -5\rangle$. What is your vector's 2nd component?

   (a) $-40/\sqrt{34}$
   (b) $40/\sqrt{34}$
   (c) $20/3$
   (d) $24/\sqrt{34}$

$\|\vec{w}\| = \sqrt{9+25} = \sqrt{34}$

$\frac{-8\vec{w}}{\|\vec{w}\|} = \frac{-24}{\sqrt{34}}\hat{i} + \frac{40}{\sqrt{34}}\hat{j}$

3. (1 point) Which one of these is a unit vector that makes a 150° angle with the positive x-axis?

   (a) $-1/2\,\hat{\imath} + \sqrt{3}/2\,\hat{\jmath}$
   (b) $\sqrt{3}/2\,\hat{\imath} - 1/2\,\hat{\jmath}$
   (c) $-\sqrt{3}/2\,\hat{\imath} + 1/2\,\hat{\jmath}$
   (d) $-\sqrt{2}/2\,\hat{\imath} + \sqrt{2}/2\,\hat{\jmath}$

$\vec{u} = \cos 150^\circ \hat{i} + \sin 150^\circ \hat{j}$

$= -\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

4. (1 point) Let M be the midpoint of $P(1,3,-9)$ and $Q(5,-1,-3)$. What is the 3rd component of the vector $\vec{MP}$?

   (a) -6
   (b) 6
   (c) -3
   (d) None of these

$M = \left(\frac{1+5}{2}, \frac{3-1}{2}, \frac{-9-3}{2}\right) = (3, 1, -6)$

$\vec{MP} = (1-3)\hat{i} + (3-1)\hat{j} + (-9-(-6))\hat{k}$

$= -2\hat{i} + 2\hat{j} - 3\hat{k}$

5. (2 points) Find the measure of angle A in triangle ABC, where $A\,(1,-1,4)$, $B\,(-2,3,1)$, and $C\,(2,2,-2)$. Which one of these is closest to your answer?

(a) 47°
(b) 36°
(c) 89°
(d) 103°

$$\vec{AB} = -3\hat{i} + 4\hat{j} - 3\hat{k}$$

$$\vec{AC} = \hat{i} + 3\hat{j} - 6\hat{k}$$

$$\cos\theta = \frac{\vec{AB}\cdot\vec{AC}}{\|\vec{AB}\|\,\|\vec{AC}\|} = \frac{-3+12+18}{\sqrt{34}\sqrt{46}} = \frac{27}{\sqrt{1564}}$$

$$\theta = \cos^{-1}\left(\frac{27}{\sqrt{1564}}\right) \approx 46.94^\circ$$

6. (2 points) True or false: The points $P\,(-2,3,3)$, $Q\,(26,-11,52)$ and $R\,(18,-7,38)$ are collinear.

(a) True
(b) False

$$\vec{PQ} = 28\hat{i} - 14\hat{j} + 49\hat{k}$$

$$\vec{PR} = 20\hat{i} - 10\hat{j} + 35\hat{k}$$

$$\vec{PQ} = \frac{7}{5}\vec{PR}$$

7. (2 points) What is the magnitude of the projection of $\vec{x} = 2\hat{i}-\hat{j}+5\hat{k}$ onto $\vec{y} = -\hat{i}+\hat{j}-7\hat{k}$?

(a) $38/\sqrt{30}$
(b) $-38/51$
(c) $38/\sqrt{51}$
(d) $19\sqrt{51}/15$

$$\text{proj}_{\vec{y}}\,\vec{x} = \frac{\vec{x}\cdot\vec{y}}{\vec{y}\cdot\vec{y}}\,\vec{y} = \frac{-38}{51}\,\vec{y}$$

$$= \frac{38}{51}\hat{i} - \frac{38}{51}\hat{j} + \frac{266}{51}\hat{k}$$

$$\|\text{proj}_{\vec{y}}\,\vec{x}\| = \frac{38}{51}\sqrt{(1)^2+(-1)^2+(7)^2}$$

$$= \frac{38}{51}\sqrt{51}$$