Math 233 - Quiz 2

January 27, 2022

Name __key__

Score _____

This quiz is available in Canvas. It is due February 1.

1. (1 point) What is the 3rd component of the projection of $\vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$ onto $\vec{a} = 3\hat{i} + 4\hat{j} - 6\hat{k}$?

   (a) 72/61

   (b) −12/61

   (c) 36/7

   (d) −6/7

   $$\text{proj}_{\vec{a}}\ \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}}\ \vec{a}$$

   $$= \frac{-12}{61} \langle 3, 4, -6 \rangle$$

2. (1 point) Let $\vec{v} = -2\hat{i} + \frac{3}{2}\hat{j} - 7\hat{k}$. For which one of the vectors below is it true that $\vec{v} \times \vec{w} = \vec{0}$?

   (a) $\vec{w} = \hat{i} + \hat{k}$

   (b) $\vec{w} = 8\hat{i} - 6\hat{j} + 28\hat{k}$

   (c) $\vec{w} = -2\hat{i} + 3\hat{j} - 7\hat{k}$

   (d) $\vec{w} = 3\hat{j}$

   $\vec{w}$ MUST BE PARALLEL TO $\vec{v}$

   $\vec{w} = -4\vec{v}$

3. (2 points) The vector $\vec{w}$ is orthogonal to both $\vec{u} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{v} = -3\hat{i} - \hat{j} + \hat{k}$. Furthermore, the 1st component of $\vec{w}$ is −2. What is the 2nd component of $\vec{w}$?

   (a) 2

   (b) −4

   (c) −8

   (d) Not enough information to tell

   $$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -3 & -1 & 1 \end{vmatrix} = \hat{i}(1) - \hat{j}(-2) + \hat{k}(5)$$

   $$= \hat{i} + 2\hat{j} + 5\hat{k}$$

   $$\vec{w} = -2\hat{i} - 4\hat{j} - 10\hat{k}$$

4. (2 points) Determine the volume of the parallelepiped determined by the vectors $\vec{x} = \langle 1, 2, -1 \rangle$, $\vec{y} = \langle 0, -1, 1 \rangle$, and $\vec{z} = \langle 1, 3, 0 \rangle$.

   (a) −2

   (b) 2

   (c) $2\sqrt{7}$

   (d) $\sqrt{62}$

   $$\begin{vmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 1 & 3 & 0 \end{vmatrix} = 1(-3) - 2(-1) + (-1)(1)$$

   $$= -3 + 2 - 1 = -2$$

   $$\text{VOLUME} = |-2| = 2$$

5. (2 points) A line passes through the points $P(2, 3, -1)$ and $Q(-5, 6, 2)$. Which one of these is an additional point on that same line?

(a) $(-2, -3, 1)$

(b) $(-7, 3, 3)$

(c) $(-3, 9, 1)$

(d) $(23, -6, -10)$

$\overrightarrow{PQ} = -7\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$

Using P...

$x = -7t + 2$
$y = 3t + 3$
$z = 3t - 1$

$t = -3$
gives
$x = 23, \quad y = -6,$
$z = -10$

6. (1 point) A line is defined by the symmetric equations $\dfrac{x+3}{2} = \dfrac{y-7}{4} = 8 - z$. Which one of these vectors is parallel to the line?

(a) $2\hat{\imath} + 4\hat{\jmath} - \hat{k}$

(b) $-3\hat{\imath} + 7\hat{\jmath} + 8\hat{k}$

(c) $2\hat{\imath} + 4\hat{\jmath} + \hat{k}$

(d) $2\hat{\imath} + 4\hat{\jmath}$

$\dfrac{x+3}{2} = \dfrac{y-7}{4} = \dfrac{z-8}{-1}$

$\vec{v} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$

7. (1 point) A line is defined by the parametric equations shown here. Which vector below is parallel to the line?

$$x = -9 + 3t$$
$$y = 7 + 2t$$
$$z = 8 - 4t$$

(a) $\langle 3, 2, 4 \rangle$

(b) $\langle 9, -7, -8 \rangle$

(c) $\langle 1, 1, 1 \rangle$

(d) $\langle 6, 4, -8 \rangle$

$\vec{v} = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$

$2\vec{v} = \langle 6, 4, -8 \rangle$