Math 233 - Quiz 3

Name __key__

February 3, 2022

Score _____

This quiz is available in Canvas. It is due February 8.

1. (1 point) Which one of these is a point on the plane $2x - 3y + 8z = 7$?

   (a) $(1/2, 6, 2)$

   (b) $(2, 1, 0)$

   (c) $(2, 7, 3)$ ⭕

   (d) None of these

$$2(2) - 3(7) + 8(3)$$
$$= 4 - 21 + 24 = 7$$

2. (2 points) Determine the angle between the planes given by $3x - 2y - 5z = 10$ and $3x + 4y + 2z = 9$.

   (a) $35.6°$

   (b) $76.0°$

   (c) $74.3°$ ⭕

   (d) None of these

$$\vec{n}_1 = 3\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$$
$$\vec{n}_2 = 3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$$
$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \|\vec{n}_2\|} = \frac{-9}{\sqrt{38}\sqrt{29}}$$

$$\theta \approx 105.7° \implies \text{Acute } \angle = 180° - 105.7°$$
$$= 74.3°$$

3. (2 points) Find a vector normal to the plane containing $\triangle ABC$, where $A(1, 2, -3)$, $B(2, 3, -5)$, and $C(4, 4, -3)$.

   (a) $4\hat{\imath} - 6\hat{\jmath} - \hat{k}$ ⭕

   (b) $-4\hat{\imath} + 6\hat{\jmath} - \hat{k}$

   (c) $4\hat{\imath} + 6\hat{\jmath} + \hat{k}$

   (d) $-4\hat{\imath} - 6\hat{\jmath} + 5\hat{k}$

$$\vec{AB} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$$
$$\vec{AC} = 3\hat{\imath} + 2\hat{\jmath}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -2 \\ 3 & 2 & 0 \end{vmatrix} = \hat{\imath}(4) - \hat{\jmath}(6) + \hat{k}(-1)$$
$$= 4\hat{\imath} - 6\hat{\jmath} - \hat{k}$$

4. (2 points) A line is described by the equations $\dfrac{x-3}{2} = \dfrac{6-y}{4} = z$. Find the distance from the line to the point $(4, 1, -2)$.

(a) 3.31

(b) 3.71

(c) 4.22

(d) 2.41

$\vec{PQ} = \langle 1, -5, -2 \rangle$

$\|\vec{V}\| = \sqrt{4+16+1} = \sqrt{21}$

$\vec{PQ} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -5 & -2 \\ 2 & -4 & 1 \end{vmatrix} = \hat{i}(-13) - \hat{j}(5) + \hat{k}(6)$

$\dfrac{\sqrt{230}}{\sqrt{21}} \approx 3.31$

$\|\vec{PQ} \times \vec{V}\| = \sqrt{169+25+36}$

$= \sqrt{230}$

5. (1 point) A surface in space is defined by the equation $x^2 - y = 1$. Which one of these is true?

(a) The surface is a cylinder with rulings parallel to the z-axis.

(b) The surface is a cylinder whose generating curve is an ellipse.

(c) The surface is a paraboloid.

(d) The surface is a hyperboloid of two sheets.

6. (1 point) A surface in space is defined by the equation $-4x^2 + 25y^2 + z^2 = 100$. Which one of these is true?

(a) The surface is a hyperboloid of one sheet.

(b) The surface is an elliptical cylinder.

(c) The surface is an elliptical cone.

(d) The surface is a hyperboloid of two sheets.

7. (1 point) A surface in space is defined by the equation $x^2 - y + z^2 = 1$. Which one of these is true?

(a) The surface is a hyperbolic paraboloid.

(b) The surface is a hyperboloid of one sheet.

(c) The surface is a circular cylinder with rulings parallel to the y-axis.

(d) The surface is a paraboloid.