Math 233 - Quiz 5 (IC)

February 24, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) A projectile is fired from 1 meter above the ground at an angle of 60° with the horizontal. The initial speed of the projectile is 158 m/sec. What is the maximum height of the projectile? (Use $g = 9.8\text{m/sec}^2$.)

$$\vec{r}(t) = 158\cos 60^\circ\, t\,\hat{i} + \left(-4.9t^2 + 158\sin 60^\circ\, t + 1\right)\hat{j}$$

$$= \underbrace{79t}_{x(t)}\,\hat{i} + \underbrace{\left(-4.9t^2 + 79\sqrt{3}\,t + 1\right)}_{y(t)}\,\hat{j}$$

$$y'(t) = -9.8t + 79\sqrt{3} = 0 \Rightarrow t = \frac{79\sqrt{3}}{9.8} \approx 13.96 \text{ sec}$$

$$y\left(\frac{79\sqrt{3}}{9.8}\right) \approx 956.26 \text{ m}$$

About 956 m.

2. (1 point) Describe the graph of a vector-valued function for which the curvature is 0 for all values of t in its domain.

The graph is a line.

Math 233 - Quiz 5 (TH)

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Score _____

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1. (4 points) A projectile is fired from ground level at an angle of 8° with the horizontal. The projectile is to have a range of 50 meters. Find the required initial velocity and the maximum height of the projectile.

$$\vec{r}(t) = v_0 \cos 8^\circ t\, \hat{i} + \left(-4.9t^2 + v_0 \sin 8^\circ t\right) \hat{j}$$

Range of 50m $\Rightarrow$ $v_0 \cos 8^\circ t = 50$

$$-4.9t^2 + v_0 \sin 8^\circ t = 0$$

$$t\left(-4.9t + v_0 \sin 8^\circ\right) = 0$$

$$t = 0 \quad \text{or} \quad t = \frac{v_0 \sin 8^\circ}{4.9}$$

$$\frac{v_0 \cos 8^\circ \, v_0 \sin 8^\circ}{4.9} = 50$$

$$v_0^2 = \frac{245}{\cos 8^\circ \sin 8^\circ} \approx 1777.7$$

$$\boxed{v_0 \approx 42.16 \text{ m/sec}}$$

Max height...

$$-9.8t + v_0 \sin 8^\circ = 0$$

$$\Rightarrow t = \frac{v_0 \sin 8^\circ}{9.8} \approx 0.59877$$

This t-value makes

$$-4.9t^2 + v_0 \sin 8^\circ t \approx \boxed{1.76 \text{ m}}$$

Turn over.

2. (3 points) An object is launched from the ground with an initial speed of 80 ft/sec and at an angle of 60° with the horizontal. Set up the definite integral that gives the length of the path of the object. Then use technology to estimate the value of the integral.

$$\vec{r}(t) = 80\cos 60^\circ t\ \hat{i} + \left(-16t^2 + 80\sin 60^\circ t\right)\hat{j}$$
$$= 40t\ \hat{i} + \left(-16t^2 + 40\sqrt{3}\,t\right)\hat{j}$$

HITS GROUND WHEN $-16t^2 + 40\sqrt{3}\,t = 0$

$$t\left(-16t + 40\sqrt{3}\right) = 0$$

$$t = 0, \quad t = \frac{40\sqrt{3}}{16} = \frac{5\sqrt{3}}{2}$$

$$\vec{r}\,'(t) = 40\,\hat{i} + \left(-32t + 40\sqrt{3}\right)\hat{j}$$

$$\text{ARC LENGTH} = \int_0^{\frac{5\sqrt{3}}{2}} \sqrt{(40)^2 + \left(-32t + 40\sqrt{3}\right)^2}\ dt$$

$$\approx 239.053 \text{ FT}$$