Math 233 - Quiz 6 (IC)

March 3, 2022

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Consider the function $f(x, y) = \ln(4 - x - y)$.

   (a) Evaluate $f(3, 0)$.

   $f(3,0) = \ln(4-3-0) = \ln(1) = \boxed{0}$

   (b) What is the domain of f?

   MUST HAVE $4-x-y > 0 \longrightarrow \boxed{\text{DOMAIN} = \{(x,y) : x+y<4\}}$

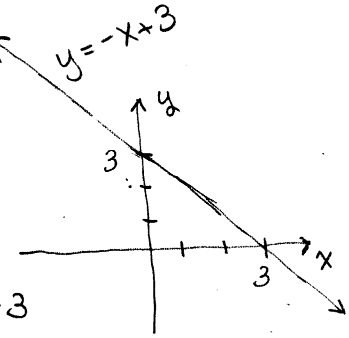
   (c) What is the range of f?

   $\boxed{\text{RANGE} = \mathbb{R}}$

   (d) Sketch the level curve $f(x, y) = 0$.

   $\ln(4-x-y) = 0 \Rightarrow 4-x-y=1$

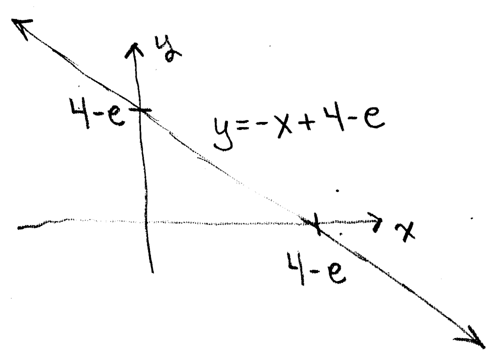
   $x+y=3 \Rightarrow y=-x+3$



   (e) Sketch the level curve $f(x, y) = 1$.

   $\ln(4-x-y) = 1 \Rightarrow 4-x-y=e$

   $x+y=4-e \Rightarrow y=-x+4-e$



   (f) Evaluate the limit: $\lim_{(x,y)\to(0,0)} f(x, y)$

   $\lim_{(x,y)\to(0,0)} \ln(4-x-y) = \boxed{\ln(4)}$

Math 233 - Quiz 6 (TH)
March 3, 2022

Name key
Score

Show all work to receive full credit. Supply explanations when necessary. This 7-point, take-home portion of the quiz is due March 8.

1. (5 points) Determine the limit or show that it does not exist.

(a) $\displaystyle\lim_{(x,y)\to(2,0)} \frac{(x-y)^2+2(x-y)-x^2-2x}{y}$ % More work.

$$= \lim_{(x,y)\to(2,0)} \frac{x^2-2xy+y^2+2x-2y-x^2-2x}{y} = \lim_{(x,y)\to(2,0)} \frac{-2xy+y^2-2y}{y}$$

$$= \lim_{(x,y)\to(2,0)} (-2x+y-2) = \boxed{-6}$$

(b) $\displaystyle\lim_{(x,y)\to(1,2)} \frac{xy^3-8x^2}{xy^2-4}$ % More work.

Let's try some paths.

Along $x=1$: $\displaystyle\lim_{y\to 2} \frac{y^3-8}{y^2-4} = \lim_{y\to 2} \frac{\cancel{(y-2)}(y^2+2y+4)}{\cancel{(y-2)}(y+2)} = \frac{12}{4} = 3$

Along $y=2$: $\displaystyle\lim_{x\to 1} \frac{8x-8x^2}{4x-4} = \lim_{x\to 1} \frac{8x(1-x)}{4(x-1)} = \lim_{x\to 1}(-2x) = -2$

Two different limits along two paths $\Rightarrow$ $\boxed{\text{Limit DNE}}$

Turn over.

2. (2 points) Let $f(x, y) = xe^{x^2+3y}$. Determine f_x and f_y.

$$\boxed{f_x(x,y) = e^{x^2+3y} + 2x^2 e^{x^2+3y}}$$ (PRODUCT RULE)

$$\boxed{f_y(x,y) = 3x\,e^{x^2+3y}}$$