Math 233 - Quiz 8
March 31, 2022

Name key

Score ____________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due April 5.

1. (2 points) Let $T = x^2y - xy^3 + 6x$, where $x = r\cos\theta$ and $y = r\sin\theta$. Use the chain rule to find $\partial T/\partial\theta$.

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial T}{\partial y}\frac{\partial y}{\partial \theta}$$

$$= (2xy - y^3 + 6)(-r\sin\theta) + (x^2 - 3xy^2)(r\cos\theta)$$

$$\frac{\partial T}{\partial \theta} = (y^3 - 2xy - 6)(r\sin\theta) + (x^2 - 3xy^2)(r\cos\theta)$$

2. (2 points) Suppose z is implicitly defined as a function of x and y by the equation

$$xy^2z^3 - xe^{yz} + y\sin xz = 3y + x.$$

Find $\partial z/\partial y$.

$$F(x,y,z) = xy^2z^3 - xe^{yz} + y\sin xz - 3y - x$$

$$\frac{\partial z}{\partial y} = \frac{-F_y}{F_z} = \frac{-(2xyz^3 - xze^{yz} + \sin xz - 3)}{3xy^2z^2 - xye^{yz} + xy\cos xz}$$

Turn over.

3. (2 points) Find the directional derivative of $f(x,y) = xe^y - ye^x$ at $(0,0)$ in the direction of $\vec{w} = 5\hat{\imath} - 2\hat{\jmath}$.

$$\vec{\nabla} f(x,y) = (e^y - ye^x)\hat{\imath} + (xe^y - e^x)\hat{\jmath}$$

$$\vec{\nabla} f(0,0) = \hat{\imath} - \hat{\jmath}$$

$$\|\vec{w}\| = \sqrt{25+4} = \sqrt{29}.$$

$$D_{\vec{w}} f(0,0) = \frac{1}{\|\vec{w}\|} \vec{\nabla} f(0,0) \cdot \vec{w}$$

$$= \frac{1}{\sqrt{29}}\left((1)(5) + (-1)(-2)\right)$$

$$= \boxed{\frac{7}{\sqrt{29}}}$$

4. (2 points) The electric potential at a point (x,y) is given by $V(x,y) = e^{-2x}\cos 3y$. Determine the direction in which the $\underbrace{\text{potential decreases most rapidly}}_{\text{OPPOSITE THE GRADIENT VECTOR.}}$ at the point $(0, \pi/4)$.

$$\vec{\nabla} V(x,y) = -2e^{-2x}\cos y\,\hat{\imath} - 3e^{-2x}\sin y\,\hat{\jmath}$$

$$\vec{\nabla} V(0, \pi/4) = (-2)(1)\left(\frac{\sqrt{2}}{2}\right)\hat{\imath} - (-3)(1)\left(\frac{\sqrt{2}}{2}\right)\hat{\jmath}$$

$$= -\sqrt{2}\,\hat{\imath} + \frac{3\sqrt{2}}{2}\hat{\jmath}$$

DIRECTION OPPOSITE THE GRADIENT IS DIRECTION OF $2\hat{\imath} - 3\hat{\jmath}$

5. (2 points) Find an equation of the plane that is ~~normal~~ TANGENT to the surface

$$xz + yz^2 = 2 + yz^3$$

at the point $(2, -1, 1)$.

$$F(x,y,z) = xz + yz^2 - 2 - yz^3$$

$$\vec{\nabla} F(x,y,z) = z\,\hat{\imath} + (z^2 - z^3)\hat{\jmath} + (x + 2yz - 3yz^2)\hat{k}$$

$$\vec{n} = \vec{\nabla} F(2,-1,1) = \hat{\imath} + 0\hat{\jmath} + 3\hat{k}$$

NORMAL LINE IS

$x = t + 2$

$y = -1$

$z = 3t + 1$

PLANE: $1(x-2) + 0(y+1) + 3(z-1) = 0$

$$\boxed{x + 3z = 5}$$