Math 233 - Quiz 9

April 7, 2022

Name __Key__

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due April 12.

1. (4 points) Find and classify the critical points of $f(x, y) = x^3 - 6xy + y^3$.

$f_x(x,y) = 3x^2 - 6y = 0 \implies y = \frac{1}{2}x^2$

$f_y(x,y) = -6x + 3y^2 = 0 \qquad -6x + \frac{3}{4}x^4 = 0 \implies x^4 - 8x = 0$

$$x(x^3 - 8) = x(x-2)(x^2 + 2x + 4)$$

$$x(x-2)(x^2 + 2x + 4) = 0$$

CRIT PTS ARE $\qquad X = 0, \quad X = 2, \quad x^2 + 2x + 4 = 0$

$(0,0) \;\&\; (2,2)$

HAS NO REAL SOLN'S.

$y = 0 \qquad y = 2$

$D(x,y) = \begin{vmatrix} 6x & -6 \\ -6 & 6y \end{vmatrix} = 36xy - 36$

$(0,0): D(0,0) = -36, \quad f(0,0) = 0 \implies \boxed{(0,0,0) \text{ IS A SADDLE PT.}}$

$(2,2): D(2,2) = 108, \quad f_{xx}(2,2) = 12 > 0 \implies \boxed{f(2,2) = -8 \text{ IS A REL MIN.}}$

Turn over.

2. (4 points) Find and classify the critical points of $g(x,y) = (x+y)(xy+1)$.

$g_x(x,y) = (1)(xy+1) + (x+y)y = y^2 + 2xy + 1 = 0$

$g_y(x,y) = (1)(xy+1) + (x+y)(x) = x^2 + 2xy + 1 = 0$

$$y^2 - x^2 = 0 \Rightarrow y = \pm x$$

$D(x,y) = \begin{vmatrix} 2y & 2y+2x \\ 2x+2y & 2x \end{vmatrix} = 4xy - (2x+2y)^2$

$y = -x \Rightarrow x^2 - 2x^2 + 1 = 0$

$1 - x^2 = 0 \Rightarrow x = \pm 1$

$(1,-1)$: $D(1,-1) = -4$, $g(1,-1) = 0$

$(1,-1)$ AND $(-1,1)$

$\boxed{(1,-1,0) \text{ IS A SADDLE PT.}}$

$y = x \Rightarrow x^2 + 2x^2 + 1 = 0$

No REAL SOLUTIONS.

$(-1,1)$: $D(-1,1) = -4$, $g(-1,1) = 0$

CRIT PTS: $(1,-1)$, $(-1,1)$

$\boxed{(-1,1,0) \text{ IS A SADDLE PT}}$

3. (2 points) Find and classify the critical points of $F(x,y) = 7y + xy - y^2 - 2x^2$.

$F_x(x,y) = y - 4x = 0 \Rightarrow y = 4x$

$F_y(x,y) = 7 + x - 2y = 0$

$7 + x - 8x = 0 \Rightarrow x = 1$

CRIT PT: $(1,4)$

$D(x,y) = \begin{vmatrix} -4 & 1 \\ 1 & -2 \end{vmatrix} = 7$

$D(1,4) = 7$, $f_{xx}(1,4) = -4 \Rightarrow \boxed{F(1,4) = 14 \text{ IS A REL MAX}}$