Test 1A

ⓘ This is a preview of the published version of the quiz

Started: Feb 16 at 9:25pm

Quiz Instructions

Choose the best answer for each problem. There is also a paper portion of the test that is posted and due Tuesday, Feb 15.

Question 1

3 pts

Which one of these vectors has magnitude 4 and makes a $210°$ angle with the positive x-axis?

$\vec{v} = 4\cos 210°\,\hat{i} + 4\sin 210°\,\hat{j}$

$= 4\left(-\frac{\sqrt{3}}{2}\right)\hat{i} + 4\left(-\frac{1}{2}\right)\hat{j}$

$= -2\sqrt{3}\,\hat{i} - 2\hat{j}$

- [x] $-2\sqrt{3}\hat{i} - 2\hat{j}$
- [ ] $-3\sqrt{2}\hat{i} - 2\hat{j}$
- [ ] $-2\sqrt{3}\hat{i} + 2\hat{j}$
- [ ] $-2\sqrt{2}\hat{i} - 2\sqrt{2}\hat{j}$

Question 2

4 pts

Determine the 2-dimensional vector of magnitude 3 that has the direction from $P\,(4, 7)$ to $Q\,(-1, 6)$. What is your vector's 2nd component?

$\vec{PQ} = -5\hat{i} - \hat{j}$, $\|\vec{PQ}\| = \sqrt{26}$

$\frac{3\vec{PQ}}{\|\vec{PQ}\|} = \frac{-15\hat{i} - 3\hat{j}}{\sqrt{26}}$

- [x] $-3/\sqrt{26}$
- [ ] $3/\sqrt{26}$
- [ ] $5/\sqrt{26}$
- [ ] $-\sqrt{3}/\sqrt{8}$

Question 3

4 pts

Let M be the midpoint of $P\,(5, 6, -3)$ and $Q\,(4, -6, 7)$. What is the 3rd component of $\overrightarrow{MP}$?

$M = \left(\frac{5+4}{2}, \frac{6+(-6)}{2}, \frac{-3+7}{2}\right) = \left(\frac{9}{2}, 0, 2\right)$

$\overrightarrow{MP} = \left(\frac{1}{2}, 6, -5\right)$

- [x] -5
- [ ] -8
- [ ] 0
- [ ] -1

Question 4

3 pts

Which vector below is NOT orthogonal to $\vec{u} = -2\hat{\imath} - 8\hat{\jmath} + 7\hat{k}$?

↑ DOT PRODUCT ZERO.

- [x] $-\frac{1}{2}\hat{\imath} - \frac{1}{8}\hat{\jmath} + \frac{2}{7}\hat{k}$
- [ ] $0\,\hat{\imath} + 0\,\hat{\jmath} + 0\,\hat{k}$
- [ ] $-\frac{3}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath} - \hat{k}$
- [ ] $4\,\hat{\imath} - \hat{\jmath}$

$-2\left(-\frac{1}{2}\right) + \left(-\frac{1}{8}\right)(-8) + 7\left(\frac{2}{7}\right)$

$= 1 + 1 + 2 = 4 \neq 0$

Question 5

6 pts

Find the area of triangle ABC, where $A\,(1, 3, -2)$, $B\,(5, 3, 1)$, and $C\,(8, -2, -3)$.

$\vec{AB} = 4\hat{i} + 0\hat{j} + 3\hat{k}$

$\vec{AC} = 7\hat{i} + -5\hat{j} - \hat{k}$

$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 3 \\ 7 & -5 & -1 \end{vmatrix} = \hat{i}(15) - \hat{j}(-25) + \hat{k}(-20)$

$= 15\hat{i} + 25\hat{j} - 20\hat{k}$

$\frac{1}{2}\|\vec{AB} \times \vec{AC}\| = \frac{1}{2}\sqrt{225 + 625 + 400}$

$= \frac{\sqrt{1250}}{2} = \frac{25\sqrt{2}}{2}$

- [x] $\frac{25\sqrt{2}}{2}$
- [ ] $\frac{\sqrt{434}}{2}$
- [ ] $\frac{5\sqrt{26}}{2}$
- [ ] $25\sqrt{2}$

Question 6

5 pts

$\vec{PQ} = -4\hat{i} - 4\hat{j}$ (Only 1st and 3rd lines have correct directions.)

A line passes through the points $P\,(1, 2, 5)$ and $Q\,(-3, 2, 1)$. Which one of these is a set of parametric equations for the line?

- [x] $x = t - 1, \quad y = 2, \quad z = t + 3$ ← P comes from $t = 2$, Q comes from $t = -2$
- [ ] $x = t - 3, \quad y = 2t + 2, \quad z = 5t + 1$
- [ ] $x = 4t + 3, \quad y = t + 2, \quad z = 4t - 1$
- [ ] $x = 4t - 1, \quad y = -2, \quad z = 4t - 5$

Question 7

3 pts

Not a hyperboloid

A surface in space is defined by the equation $-4x^2 + 16y^2 - 8z^2 = 0$. Which one of these is true?

Fix $y = k \Rightarrow 4x^2 + 8z^2 = 16k^2$ (ellipses)

- [ ] The surface is a cylinder with an elliptical generating curve.
- [x] The surface is a cone whose traces parallel to the xz-plane are ellipses.
- [ ] The surface is a cone whose traces parallel to the xy-plane are parabolas.
- [ ] The surface is a hyperboloid of one sheet.

Cross sections are Hyperbolas, Ellipses, Hyperbolas

Question 8 — 3 pts

A surface in space is defined by the equation $x^2 + z^2 = 4$. Which one of these is true?

CIRCLES UP THE y-AXIS

- [x] The surface is a circular cylinder with rulings parallel to the y-axis.
- [ ] The surface is a paraboloid with circular cross sections.
- [ ] The surface is a cylinder whose generating curve is a parabola.
- [ ] The surface is a cone whose traces parallel to the xz-plane are circles.

Question 9 — 3 pts

A surface in space is defined by the equation $x^2 - y^2 + z^2 + 5 = 0$. Which one of these is true?

CROSS SECTIONS ARE HYPERBOLAS, CIRCLES, HYPERBOLAS.

- [x] The surface is a hyperboloid of two sheets.
- [ ] The surface is a hyperboloid on one sheet.
- [ ] The surface is a cone.
- [ ] The surface is a sphere.

FIX $y=k$: $x^2+z^2 = k^2-5$

↑ NO CIRCLE IF k IS SMALL

Question 10 — 6 pts

Compute the principal unit tangent vector at $t = \pi/2$.

$\vec{r}(t) = \sin(2t)\,\hat{\imath} + 2\cos(t)\,\hat{\jmath} + t\,\hat{k}$

$\vec{r}\,'(t) = 2\cos 2t\,\hat{\imath} - 2\sin t\,\hat{\jmath} + \hat{k}$

- [ ]

- [x] $-\frac{2}{3}\hat{i}-\frac{2}{3}\hat{j}+\frac{1}{3}\hat{k}$
- [ ] $\frac{2}{3}\hat{i}+\frac{2}{3}\hat{j}-\frac{1}{3}\hat{k}$
- [ ] $-\frac{1}{\sqrt{6}}\hat{i}-\frac{2}{\sqrt{6}}\hat{j}+\frac{1}{\sqrt{6}}\hat{k}$
- [ ] $\frac{1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}+\frac{1}{\sqrt{6}}\hat{k}$

$\vec{r}'\left(\frac{\pi}{2}\right) = -2\hat{i} - 2\hat{j} + \hat{k}$

$\|\vec{r}'\left(\frac{\pi}{2}\right)\| = \sqrt{4+4+1} = 3$

$\hat{T}\left(\frac{\pi}{2}\right) = \frac{\vec{r}'\left(\frac{\pi}{2}\right)}{\|\vec{r}'\left(\frac{\pi}{2}\right)\|} = -\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$

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