Math 233 - Test 1B
February 10, 2022

Name key Score ________

Show all work to receive full credit. Supply explanations where necessary. This test is due February 15. The other portion of the test is in Canvas. **You must work individually on this test.**

1. (5 points) The addition of vectors is often described by the phrase "triangle method" or "parallelogram method." Sketch the vectors $\vec{u} = \langle 2, 3 \rangle$ and $\vec{w} = \langle -3, 1 \rangle$. Then illustrate $\vec{u} + \vec{w}$ by using either method.

SEE ATTACHED SHEETS.

2. (8 points) Find an equation the plane that passes through the points $A\,(4, 0, -5)$, $B\,(8, -3, -6)$, and $C\,(1, 2, -1)$. Write your final answer in standard form.

$\vec{AB} = 4\hat{\imath} - 3\hat{\jmath} - \hat{k}$
$\vec{AC} = -3\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

$$\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -3 & -1 \\ -3 & 2 & 4 \end{vmatrix} = \hat{\imath}(-10) - \hat{\jmath}(13) + \hat{k}(-1)$$
$$= -10\hat{\imath} - 13\hat{\jmath} - \hat{k}$$

I'LL USE $\vec{n} = 10\hat{\imath} + 13\hat{\jmath} + \hat{k}$.

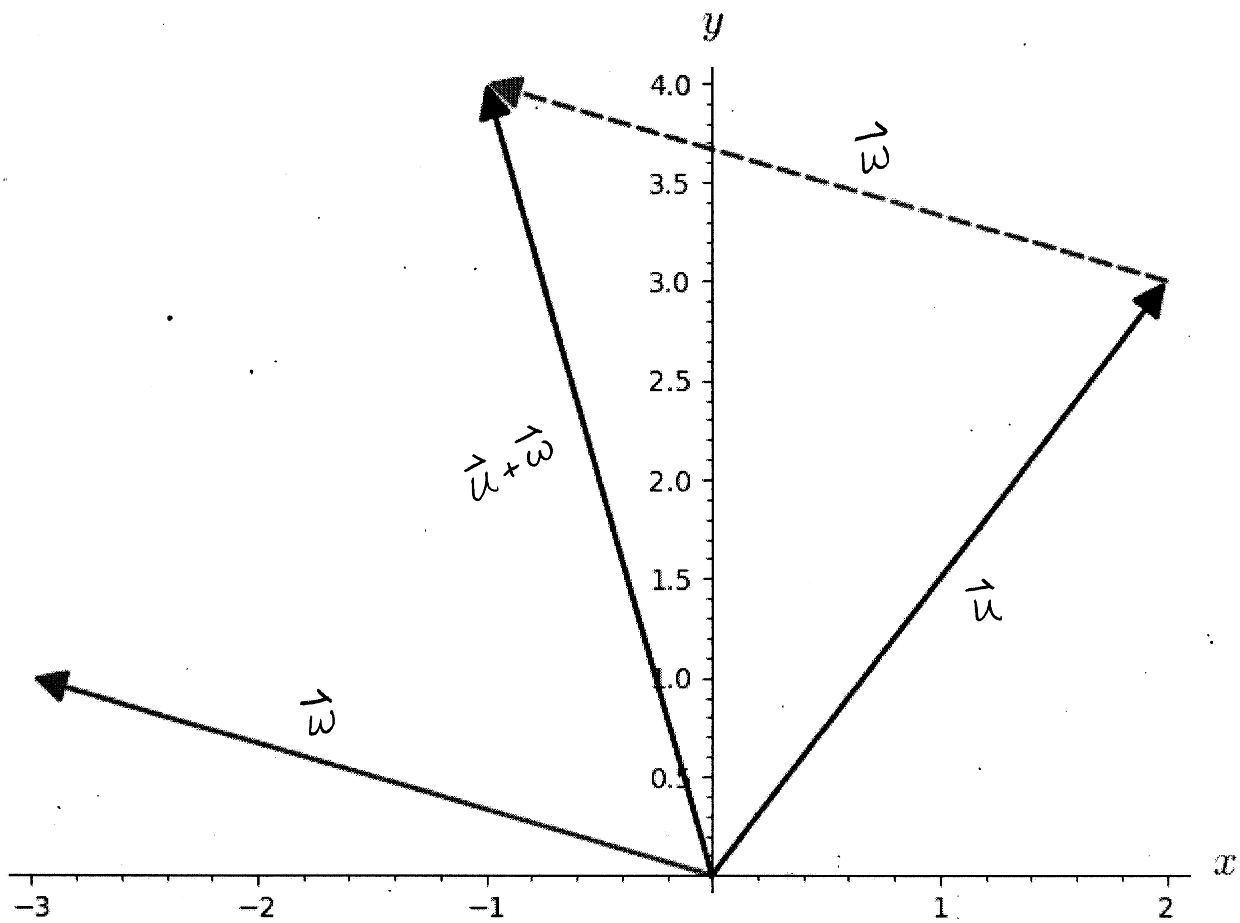
PLANE IS $10(x-4) + 13(y-0) + 1(z+5) = 0$

OR

$$\boxed{10x + 13y + z - 35 = 0 \quad \text{OR} \quad 10x + 13y + z = 35}$$

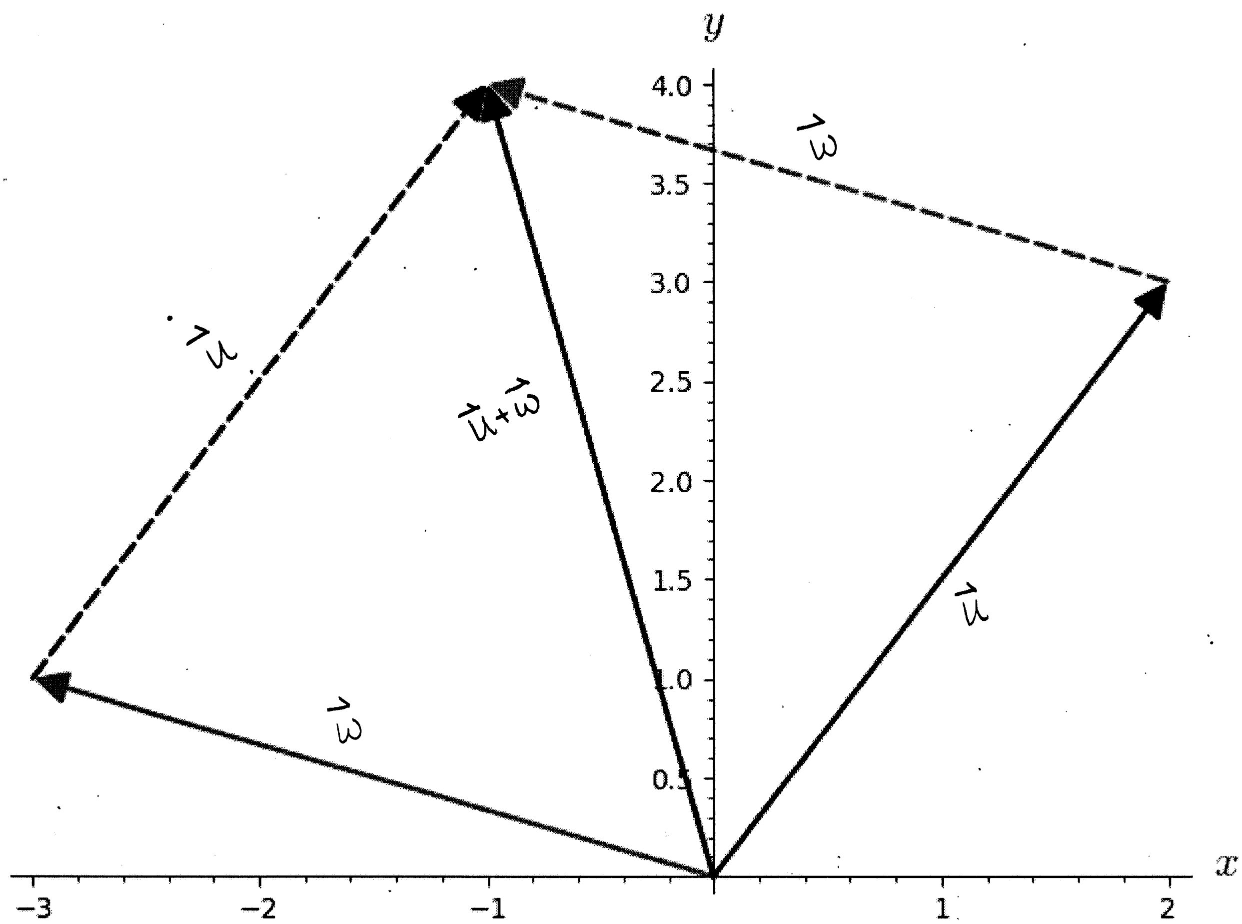
#1

TRIANGLE METHOD...



#1

PARALLELOGRAM METHOD ...



3. (4 points) Determine the vector of magnitude 7 that has the opposite direction of $\vec{v} = -4\hat{\imath} + 13\hat{\jmath} - 16\hat{k}$.

$\|\vec{v}\| = \sqrt{16 + 169 + 256}$

$= 21$

$\dfrac{-7\vec{v}}{\|\vec{v}\|} = \dfrac{-7}{21}\langle -4, 13, -16 \rangle$

$= \boxed{\frac{4}{3}\hat{\imath} - \frac{13}{3}\hat{\jmath} + \frac{16}{3}\hat{k}}$

4. (8 points) Let $\vec{u} = -4\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ and $\vec{v} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$.

(a) Let $\vec{w} = \text{proj}_{\vec{v}}\,\vec{u}$. Compute $\vec{w}$.

$\text{proj}_{\vec{v}}\,\vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\vec{v} = \dfrac{-4-8-6}{1+4+4}\vec{v} = \dfrac{-18}{9}\vec{v} = -2\vec{v}$

$\boxed{\vec{\omega} = -2\hat{\imath} + 4\hat{\jmath} - 4\hat{k}}$

(b) Compute $\vec{u} - \vec{w}$ and show that $\vec{u} - \vec{w}$ is orthogonal to $\vec{w}$.

$\vec{u} - \vec{\omega} = -2\hat{\imath} + 0\hat{\jmath} + \hat{k}$

$= -2\hat{\imath} + \hat{k}$

$(\vec{u} - \vec{\omega}) \cdot \vec{\omega} = \langle -2, 0, 1 \rangle \cdot \langle -2, 4, -4 \rangle$

$= 4 + 0 - 4 = 0$

DOT PROD. ZERO $\Rightarrow$ VECTORS $\therefore$ ARE ORTHOG.

5. (8 points) Determine the measure of the acute angle between the planes whose equations are shown below. Write your answer in degrees, rounded to the nearest hundredth.

$2x - y - 2z = 12$ $\qquad\qquad$ $3x + 2y + 4z = -9$

$\vec{n}_1 = 2\hat{\imath} - \hat{\jmath} - 2\hat{k}$ $\qquad\qquad$ $\vec{n}_2 = 3\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

θ = ACUTE ANGLE BETWEEN PLANES

$\cos\theta = \dfrac{|\vec{n}_1\cdot\vec{n}_2|}{\|\vec{n}_1\|\|\vec{n}_2\|} = \dfrac{|6-2-8|}{\sqrt{9}\sqrt{29}} = \dfrac{|-4|}{\sqrt{261}} = \dfrac{4}{\sqrt{261}}$

$\boxed{\theta \approx 75.66^\circ}$

6. (8 points) The line L_1 has parametric equations $x = 6t - 7$, $y = -2t - 3$, $z = t - 3$. The line L_2 has symmetric equations $\frac{x-2}{3} = \frac{y}{4} = z$.

(a) Show that L_1 and L_2 are not parallel.

$\vec{V}_1 \neq k\vec{V}_2$.

$\vec{V}_1 = 6\hat{i} - 2\hat{j} + \hat{k}$ = DIRECTION OF L_1

$\vec{V}_2 = 3\hat{i} + 4\hat{j} + \hat{k}$ = DIRECTION OF L_2

$\vec{V}_1$ IS NOT A SCALAR MULTIPLE OF $\vec{V}_2$.

(b) Show that L_1 and L_2 do not intersect.

PARAMETRIC EQUATIONS FOR L_2:

$x = 3s + 2$
$y = 4s$
$z = s$

AT A POINT OF INTERSECTION, WE MUST HAVE
$3s + 2 = 6t - 7$ ← $s = -1.5$ AND $t = 1.5$ DO NOT SOLVE

$$\left.\begin{aligned} 4s &= -2t - 3 \\ s &= t - 3 \end{aligned}\right\}$$ THESE ARE SOLVED WHEN $s = -1.5$ AND $t = 1.5$

NO SUCH s & t!

7. (9 points) Let $\vec{r}(t) = \left(\frac{2t-2}{t^2-1}\right)\hat{\imath} - \tan^{-1}(t)\,\hat{\jmath} + \ln(2t)\,\hat{k}$.

(a) Determine the domain of $\vec{r}$.

$x(t) = \frac{2t-2}{t^2-1}$, DOMAIN: $t \neq \pm 1$

$y(t) = -\tan^{-1} t$, DOMAIN: $\mathbb{R}$

$z(t) = \ln(2t)$, DOMAIN: $2t > 0$ OR $t > 0$

⇒ DOMAIN OF $r(t)$: $t > 0$ & $t \neq 1$

(b) Compute $\lim_{t\to 1} \vec{r}(t)$.

$$\lim_{t\to1} \vec{r}(t) = \left(\lim_{t\to1} \frac{2\cancel{(t-1)}}{\cancel{(t-1)}(t+1)}\right)\hat{i} - \left(\lim_{t\to1} \tan^{-1} t\right)\hat{j} + \left(\lim_{t\to1} \ln 2t\right)\hat{k}$$

$$= \frac{2}{2}\hat{i} - \tan^{-1}(1)\,\hat{j} + \ln 2\,\hat{k} = \boxed{\hat{i} - \frac{\pi}{4}\hat{j} + (\ln 2)\hat{k}}$$

(c) Compute $\frac{d\vec{r}}{dt}$.

$$\vec{r}(t) = \frac{2}{t+1}\hat{i} - \tan^{-1}(t)\,\hat{j} + \ln(2t)\,\hat{k}, \quad t \neq 1$$

$$\vec{r}\,'(t) = -\frac{2}{(t+1)^2}\hat{i} - \frac{1}{1+t^2}\hat{j} + \frac{1}{t}\hat{k}, \quad t \neq 1$$

8. (5 points) Sketch the graph of the vector-valued function $\vec{r}(t) = (2t-1)^2\,\hat{\imath} + (2t+2)\,\hat{\jmath}$. Draw arrows on your graph to indicate the orientation.

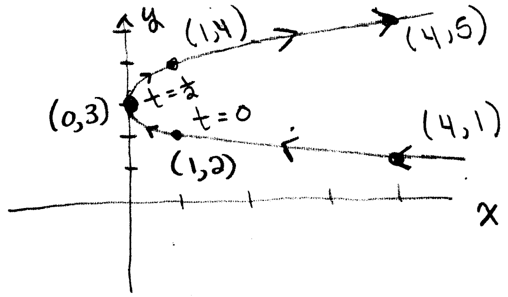
$x = (2t-1)^2$

$y = 2t+2$

$t = \frac{y-2}{2}$

$x = (y-3)^2$

GRAPH IS A PARABOLA, OPENING TOWARD THE RIGHT, WITH VERTEX AT $(0,3)$.



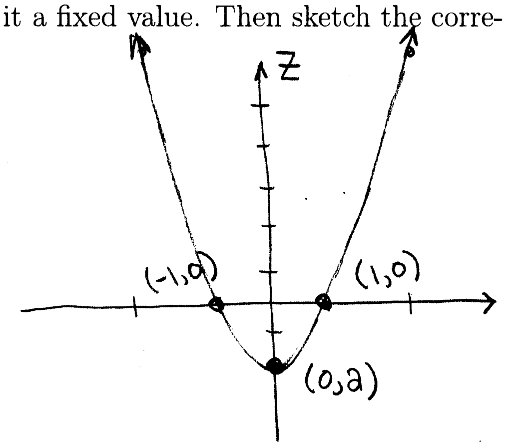
SEE ATTACHED SHEET FOR BETTER GRAPH.

9. (5 points) Consider the surface defined by the equation $2x^2 - 2y^2 - z = 0$.

(a) Choose any one of the variables and give it a fixed value. Then sketch the corresponding trace.

$y = 1 \Rightarrow \quad z = 2x^2 - 2$

PARABOLA, VERTEX AT $(0,-2)$, OPENING UPWARD.



(b) Identify the surface. Very briefly explain how you know.

THE SURFACE IS A HYPERBOLIC PARABOLOID.

FIX X	FIX Y	FIX $Z \neq 0$
TRACES ARE PARABOLAS.	TRACES ARE PARABOLAS.	TRACES ARE HYPERBOLAS.

#8