Math 233 - Test 1B

February 10, 2022

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. This test is due February 15. The other portion of the test is in Canvas. **You must work individually on this test.**

1. (5 points) The addition of vectors is often described by the phrase "triangle method" or "parallelogram method." Sketch the vectors $\vec{u} = \langle 2, 3 \rangle$ and $\vec{w} = \langle -3, 1 \rangle$. Then illustrate $\vec{u} + \vec{w}$ by using either method.

2. (8 points) Find an equation the plane that passes through the points $A\,(4, 0, -5)$, $B\,(8, -3, -6)$, and $C\,(1, 2, -1)$. Write your final answer in standard form.

3. (4 points) Determine the vector of magnitude 7 that has the opposite direction of $\vec{v} = -4\hat{\imath} + 13\hat{\jmath} - 16\hat{k}$.

4. (8 points) Let $\vec{u} = -4\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ and $\vec{v} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$.

   (a) Let $\vec{w} = \text{proj}_{\vec{v}}\,\vec{u}$. Compute $\vec{w}$.

   (b) Compute $\vec{u} - \vec{w}$ and show that $\vec{u} - \vec{w}$ is orthogonal to $\vec{w}$.

5. (8 points) Determine the measure of the acute angle between the planes whose equations are shown below. Write your answer in degrees, rounded to the nearest hundredth.

$$2x - y - 2z = 12 \qquad\qquad 3x + 2y + 4z = -9$$

6. (8 points) The line L_1 has parametric equations $x = 6t - 7$, $y = -2t - 3$, $z = t - 3$. The line L_2 has symmetric equations $\dfrac{x-2}{3} = \dfrac{y}{4} = z$.

(a) Show that L_1 and L_2 are not parallel.

(b) Show that L_1 and L_2 do not intersect.

7. (9 points) Let $\vec{r}(t) = \left(\dfrac{2t-2}{t^2-1}\right)\hat{\imath} - \tan^{-1}(t)\,\hat{\jmath} + \ln(2t)\,\hat{k}$.

(a) Determine the domain of $\vec{r}$.

(b) Compute $\lim\limits_{t\to 1} \vec{r}(t)$.

(c) Compute $\dfrac{d\vec{r}}{dt}$.

8. (5 points) Sketch the graph of the vector-valued function $\vec{r}(t) = (2t-1)^2\,\hat{\imath} + (2t+2)\,\hat{\jmath}$. Draw arrows on your graph to indicate the orientation.

9. (5 points) Consider the surface defined by the equation $2x^2 - 2y^2 - z = 0$.

   (a) Choose any one of the variables and give it a fixed value. Then sketch the corresponding trace.

   (b) Identify the surface. Very briefly explain how you know.