

Math 233 - Test 2

March 10, 2022

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (9 points) A curve is defined by the vector-valued function shown below. Starting from $t = 0$, reparameterize the curve in terms of the arc-length parameter.

$$\vec{r}(t) = (7t - 6)\hat{i} + (4t + 1)\hat{j} + (4t + 3)\hat{k}.$$

Follow-up: Once you have reparameterized, compute $\|\vec{r}'(s)\|$.

2. (5 points) Sketch, or describe in detail, a 2-dimensional curve whose curvature is constant and nonzero. Then say what the curvature of your curve actually is and how you know.

3. (16 points) A cannonball is fired toward the sea from a 192-ft cliff. The cannon is aimed at an angle of 30° above the horizontal and the initial speed of the cannonball is 128 ft/sec. (Use $g = 32 \text{ ft/sec}^2$ for this problem.)

(a) Determine the function that gives the position of the cannonball at time t .

(b) When will the cannonball splash into the sea?

(c) How far out to sea will the cannonball hit the water?

(d) Set up the definite integral that gives the total length of the cannonball's path. Use your calculator to estimate the value of the integral.

4. (8 points) Consider the function $f(x, y) = \ln(4 - x - y)$.
- (a) Evaluate $f(2, 1)$.

 - (b) What is the domain of f ?

 - (c) What is the range of f ?

 - (d) Sketch the level curve $f(x, y) = 0$.

 - (e) Sketch the level curve $f(x, y) = 1$.
5. (8 points) Let $G(x, y, z) = \sqrt{x^2 + y^2 - z}$.
- (a) Evaluate $G(-4, 5, -8)$.

 - (b) What is the domain of G ?

 - (c) Describe or sketch the level surface $G(x, y, z) = 0$.

 - (d) Describe or sketch the level surface $G(x, y, z) = -1$.

 - (e) Describe or sketch the level surface $G(x, y, z) = 1$.

6. (20 points) Determine the limit or show that it does not exist.

$$(a) \lim_{(x,y) \rightarrow (3,3)} \frac{x^2 - y^2 + x - y}{2x - 2y}$$

$$(b) \lim_{(x,y,z) \rightarrow (1,2,-1)} \frac{2x - 3y - 4z}{x + y^2 - z}$$

$$(c) \lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$$

$$(d) \lim_{(x,y) \rightarrow (1,1)} \frac{x - y}{\sqrt{x} - \sqrt{y}}$$

7. (8 points) Let $f(x, y, z) = x^2y^3 + 2xyz - 3yz$.

(a) Compute $f_x(-2, 1, 2)$.

(b) Which is (slightly) simpler to compute f_{xy} or f_{yx} ? Why?

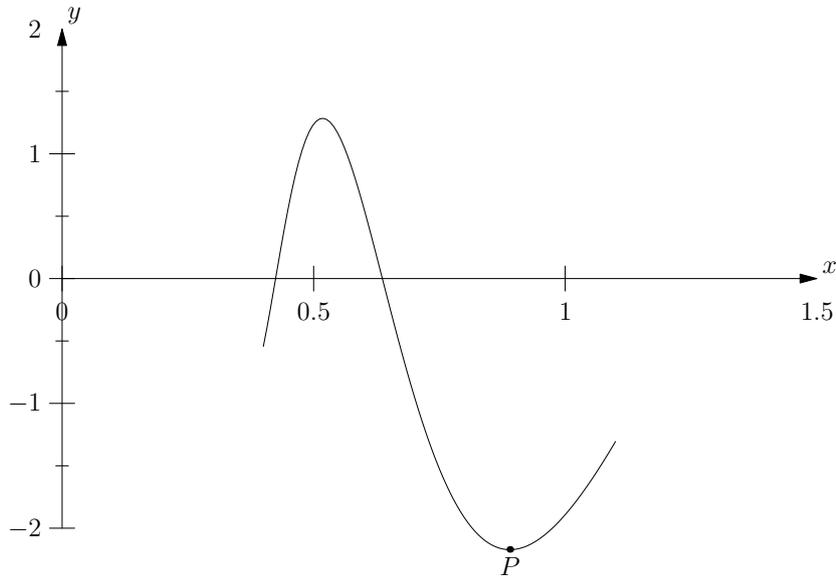
(c) Do you expect that $f_{xy}(x, y) = f_{yx}(x, y)$? Why?

(d) Compute $f_{yz}(x, y, z)$.

8. (5 points) Consider the function $f(x, y) = (x^2 + y^2)^{1/3}$. Would you expect $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ to be equal? Explain your reasoning.

9. (5 points) If you stood on the graph of $z = \ln(xy^2 - y + 1)$ at the point $(x, y) = (1, -1)$ and looked in the direction of the positive y -axis, would you be looking uphill or downhill? Show your work.

10. (6 points) Suppose a particle moves along the curve from **right to left**. Sketch and label each of the following. Make note of the scale.
- (a) The unit tangent vector at the point of greatest curvature
 - (b) A point where the principal unit normal vector does not exist
 - (c) The principal unit normal vector at the point P
 - (d) (1 pt ex cred) The circle of curvature at the P



11. (10 points) Let $\vec{r}(t) = \sin 3t \hat{i} - \cos 3t \hat{j} + 3t \hat{k}$. Compute $\hat{N}(t)$.