

Math 233 - Test 3a
April 14, 2022

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) The volume V of a right circular cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. Suppose that the height decreases from 20 in to 19.95 in, while the radius increases from 4 in to 4.06 in. Use differentials to approximate the change in volume.

2. (6 points) Find the directional derivative of $f(x, y) = \tan^{-1}(y/x)$ at the point $(-2, 2)$ in the direction of $\vec{v} = -\hat{i} - \hat{j}$.

3. (6 points) Let $z = x^2y^2 - x + 2y$, where $x = \sqrt{u}$ and $y = uv^3$. Use the appropriate chain rule to compute $\partial z/\partial u$ when $(u, v) = (1, -2)$.

4. (6 points) Let $G(x, y, z) = \frac{x}{z} + \frac{z}{y^2}$. Find a unit vector in the direction in which G decreases most rapidly at $P(1, 2, -2)$. What is the corresponding rate of decrease?

5. (6 points) Find the linearization of $f(x, y) = x^{1/3}y^{1/2}$ at $(x, y) = (8, 9)$. Then use your linearization to approximate $f(7.87, 9.09)$.

6. (8 points) Find an equation of the plane tangent to the surface $\sin(xz) = 4 \cos(yz)$ at the point $(\pi, \pi/2, 1)$.

7. (6 points) Find and classify the critical point(s) of $f(x, y) = x^2 + xy + y^2 - 3x$.

8. (10 points) Consider the iterated integral $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3} dx dy$. Sketch the region of integration, reverse the order of integration, and evaluate.

9. (6 points) Assume that the equation $x^3 + y^2x - 3 = 0$ implicitly defines y as a function of x . Use partial derivatives to find dy/dx .