Math 233 - Final Exam A Name key

May 5, 2022 Score ________

Show all work to receive full credit. Each problem is worth 5 points–up to 2 points for the answer and up to 3 points for the supporting work or explanation. Place your final answer in the box provided. This test is due May 10. You must work individually.

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1. The planes defined by the equations

$$x + 2y - 2z = 3 \qquad \text{and} \qquad 2x + 4y - 4z = 7$$

are parallel. Find a point on the first plane and find the distance between the planes.

BY OBSERVATION, $(3,0,0)$ IS A POINT ON FIRST PLANE: $(3)+2(0)-2(0)=3$ ✓

$$d = \frac{|2(3)+4(0)-4(0)-7|}{\sqrt{4+16+16}} = \frac{1}{\sqrt{36}} = \frac{1}{6}$$

$$\boxed{\text{POINT } (3,0,0), \quad \text{DISTANCE} = \frac{1}{6}}$$

2. Find the limit or show that it does not exist: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{5x^3y}{2x^6+y^2}$

ALONG $x=0$: $\displaystyle\lim_{y\to 0} \frac{0}{y^2} = \lim_{y\to 0} 0 = 0$

ALONG $y=x^3$: $\displaystyle\lim_{x\to 0} \frac{5x^6}{3x^6} = \lim_{x\to 0} \frac{5}{3} = \frac{5}{3}$

LIMIT DNE BY TWO-PATH TEST.

$$\boxed{\text{LIMIT DNE}}$$

$\vec{r}(t) = 800\cos\theta\, t\, \hat{\imath} + (-16t^2 + 800\sin\theta\, t)\,\hat{\jmath}$

3. Find two elevation angles that will enable a shell, fired from ground level with a muzzle speed of 800 ft/sec, to hit a ground-level target 10,000 feet away. Write your final answers in degrees. (Ignore air resistance and use $g \approx 32\,\text{ft/sec}^2$.)

$800\cos\theta\, t = 10000$

$-16t^2 + 800\sin\theta\, t = 0 \Rightarrow -16t(t - 50\sin\theta) = 0$

$t = 50\sin\theta$

$800(50\cos\theta\sin\theta) = 10000$

$800(25)(2\cos\theta\sin\theta) = 10000$

$\sin 2\theta = \frac{1}{2}$

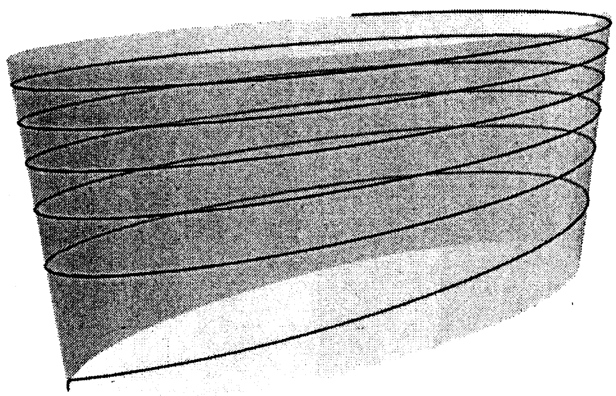
$2\theta = 30°, 150°$

$$\boxed{\theta = 15° \quad \text{or} \quad \theta = 75°}$$

4. A wire is wrapped around an elliptical steel tube so that the wire has the shape of the graph of

$$\vec{r}(t) = 6\cos(t)\,\hat{\imath} + 2\sin(t)\,\hat{\jmath} + \sqrt{t}\,\hat{k}, \quad 0 \le t \le 36,$$

where $\vec{r}$ is in centimeters. Set up the definite integral that gives the length of the wire. Use technology to approximate the value of your integral.



$\vec{r}\,'(t) = -6\sin t\,\hat{\imath} + 2\cos t\,\hat{\jmath} + \frac{1}{2}t^{-1/2}\,\hat{k}$

$\|\vec{r}\,'(t)\| = \left(36\sin^2 t + 4\cos^2 t + \frac{1}{4t}\right)^{1/2}$

$$\boxed{\int_0^{36}\left(36\sin^2 t + 4\cos^2 t + \frac{1}{4t}\right)^{1/2} dt \approx 153.3\text{ cm}}$$

Improper at $t = 0$

5. Nonlinear systems of equations are often solved numerically by "linearizing" the equations. Suppose you want to solve the system

$$f(x, y) = x^2 + xy^3 - 9 = 0 \qquad g(x, y) = 3x^2y - y^3 - 4 = 0.$$

Find the linearization of each function at $(x, y) = (-1.2, 2.5)$.

$f(-1.2, 2.5) = -26.31$

$f_x(x,y) = 2x + y^3$, $f_x(-1.2, 2.5) = 13.225$

$f_y(x,y) = 3xy^2$, $f_y(-1.2, 2.5) = -22.5$

$g(-1.2, 2.5) = -8.825$

$g_x(x,y) = 6xy$, $g_x(-1.2, 2.5) = -18$

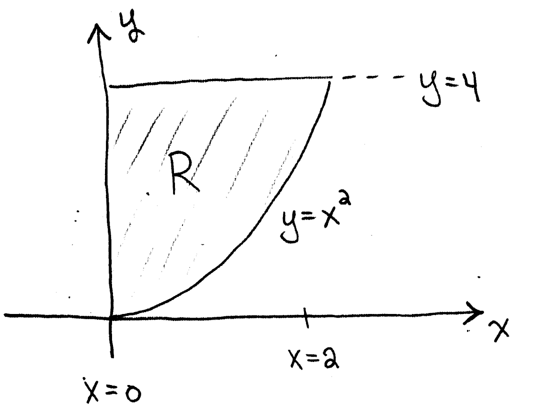
$g_y(x,y) = 3x^2 - 3y^2$, $g_y(-1.2, 2.5) = -14.43$

For f: $L_1(x,y) = -26.31 + 13.225(x+1.2) - 22.5(y-2.5)$

For g: $L_2(x,y) = -8.825 - 18(x+1.2) - 14.43(y-2.5)$

6. Evaluate the iterated integral by first reversing the order of integration. Do not use technology to evaluate the integral.

$$\int_0^2 \int_{x^2}^4 \frac{x^3}{\sqrt{x^4 + y^2}}\, dy\, dx$$



$$\int_{y=0}^{y=4} \int_{x=0}^{x=\sqrt{y}} \frac{x^3}{\sqrt{x^4+y^2}}\, dx\, dy = \frac{1}{4}\int_0^4 \int_{y^2}^{2y^2} \frac{1}{\sqrt{u}}\, du\, dy$$

$u = x^4 + y^2$

$du = 4x^3\, dx$

$$= \frac{1}{2}\int_0^4 \sqrt{u}\,\Big|_{y^2}^{2y^2} dy$$

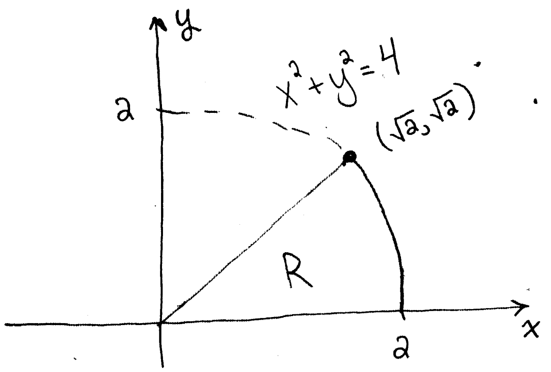
$$\boxed{\int_0^4 \int_0^{\sqrt{y}} \frac{x^3}{\sqrt{x^4+y^2}}\, dx\, dy = 4(\sqrt{2}-1)}$$

$$\frac{1}{2}\int_0^4 \sqrt{2}y - y\, dy = \frac{\sqrt{2}-1}{2}\int_0^4 y\, dy = \frac{\sqrt{2}-1}{2}\left(\frac{16}{2}\right)$$

7. Convert the iterated integral to an equivalent integral in cylindrical coordinates. Then use technology to evaluate the integral.

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} x^2\, dz\, dx\, dy$$

R



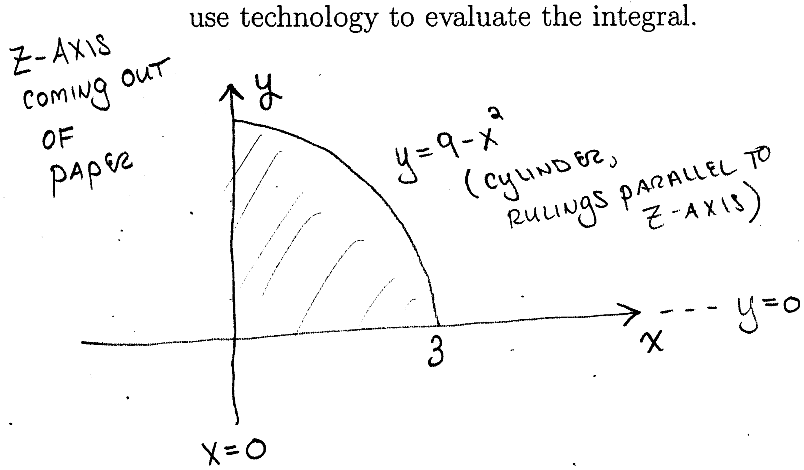
$$\int_{\theta=0}^{\theta=\frac{\pi}{4}} \int_{r=0}^{r=2} \int_{z=r}^{z=\sqrt{8-r^2}} r^3 \cos^2\theta \, dz\, dr\, d\theta$$

$$= \frac{8}{15}(\pi+2)(4\sqrt{2}-5)$$

$$\approx 1.8012$$

$$\boxed{\int_0^{\pi/4} \int_0^{2} \int_r^{\sqrt{8-r^2}} r^3 \cos^2\theta \, dz\, dr\, d\theta \approx 1.8012}$$

8. A solid lies in the 1st octant bounded by the surface $y = 9 - x^2$ (where $x \geq 0$) and the planes $y = 0$, $z = 0$, and $z = 1$. Its density at the point (x, y, z) is given by $\rho(x, y, z) = 1 + xyz$. Set up the iterated integral that gives the mass of the solid. Then use technology to evaluate the integral.



$$\text{MASS} = \int_{x=0}^{x=3} \int_{y=0}^{y=9-x^2} \int_{z=0}^{z=1} (1+xyz)\, dz\, dy\, dx$$

$$= \frac{387}{8}$$

$$= 48.375$$

$$\boxed{\int_0^3 \int_0^{9-x^2} \int_0^1 (1+xyz)\, dz\, dy\, dx = \frac{387}{8}}$$