

# Math 233 - Final Exam A

May 5, 2022

Name \_\_\_\_\_

Score \_\_\_\_\_

**Show all work to receive full credit.** Each problem is worth 5 points—up to 2 points for the answer and up to 3 points for the supporting work or explanation. Place your final answer in the box provided. This test is due May 10. You must work individually.

---

1. The planes defined by the equations

$$x + 2y - 2z = 3 \quad \text{and} \quad 2x + 4y - 4z = 7$$

are parallel. Find a point on the first plane and find the distance between the planes.

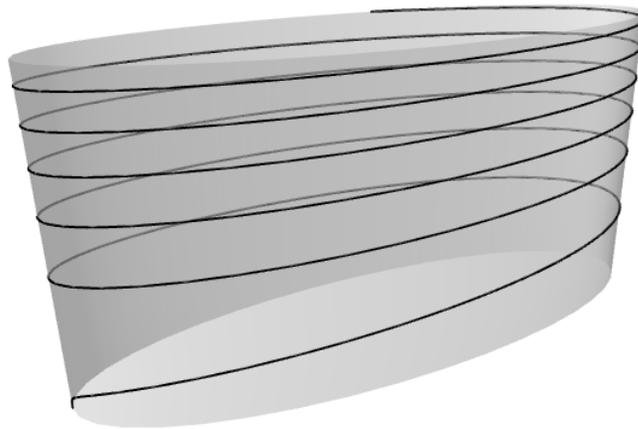
2. Find the limit or show that it does not exist:  $\lim_{(x,y) \rightarrow (0,0)} \frac{5x^3y}{2x^6 + y^2}$

3. Find two elevation angles that will enable a shell, fired from ground level with a muzzle speed of 800 ft/sec, to hit a ground-level target 10,000 feet away. Write your final answers in degrees. (Ignore air resistance and use  $g \approx 32 \text{ ft/sec}^2$ .)

4. A wire is wrapped around an elliptical steel tube so that the wire has the shape of the graph of

$$\vec{r}(t) = 6 \cos(t) \hat{i} + 2 \sin(t) \hat{j} + \sqrt{t} \hat{k}, \quad 0 \leq t \leq 36,$$

where  $\vec{r}$  is in centimeters. Set up the definite integral that gives the length of the wire. Use technology to approximate the value of your integral.



5. Nonlinear systems of equations are often solved numerically by “linearizing” the equations. Suppose you want to solve the system

$$f(x, y) = x^2 + xy^3 - 9 = 0 \quad g(x, y) = 3x^2y - y^3 - 4 = 0.$$

Find the linearization of each function at  $(x, y) = (-1.2, 2.5)$ .

6. Evaluate the iterated integral by first reversing the order of integration. Do not use technology to evaluate the integral.

$$\int_0^2 \int_{x^2}^4 \frac{x^3}{\sqrt{x^4 + y^2}} dy dx$$

7. Convert the iterated integral to an equivalent integral in cylindrical coordinates. Then use technology to evaluate the integral.

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} x^2 dz dx dy$$

8. A solid lies in the 1st octant bounded by the surface  $y = 9 - x^2$  (where  $x \geq 0$ ) and the planes  $y = 0$ ,  $z = 0$ , and  $z = 1$ . Its density at the point  $(x, y, z)$  is given by  $\rho(x, y, z) = 1 + xyz$ . Set up the iterated integral that gives the mass of the solid. Then use technology to evaluate the integral.