Math 233 - Final Exam B

Name _____

May 12, 2022

Score _____

Show all work to receive full credit. Each problem is worth 5 points–up to 2 points for the answer and up to 3 points for the supporting work or explanation. Place your final answer in the box provided.

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1. A particle is moved from the point $A\,(0,2,1)$ to the point $B\,(3,1,6)$ by applying the force $\vec{F} = 4\hat{\imath} + \hat{\jmath} + 3\hat{k}$. Find the projection of $\vec{F}$ onto $\vec{AB}$.

2. Find the volume of the parallelepiped determined by the vectors $\vec{u} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$, $\vec{v} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, and $\vec{w} = \hat{\jmath} - 2\hat{k}$.

3. The line L_1 has symmetric equations

$$\frac{x-3}{2} = \frac{2-y}{3} = \frac{z}{7}.$$

The line L_2 is parallel to L_1 and passes through the point $(1, 2, -3)$. Find a set of parametric equations for L_2.

4. Let $G(x, y, z) = \sqrt{x^2 + y^2 - z}$. Describe the level surface $G(x, y, z) = 1$.

5. Find the limit or show that it does not exist: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$

6. The temperature at a point in a solid is given by

$$T(x, y, z) = \frac{xyz}{1 + x^2 + y^2 + z^2}.$$

Use differentials to estimate the change in temperature from the point $(1, 1, 1)$ to the point $(1.05, -0.98, 1.02)$.

7. Find an equation of the plane tangent to graph of $\sin(xz) - 4\cos(yz) = 4$ at the point $(\pi, \pi, 1)$.

8. On a certain mountain, the elevation z above the point (x, y) is given by

$$z = 2000 - 2x^2 - 4y^2,$$

where the coordinates are measured in meters. Assume that the positive x-axis points east, and the positive y-axis points north. A climber at the point $(-20, 5, 1100)$ uses a compass reading to walk northeast. Will the climber ascend or descend? At what rate?

9. Find the critical point(s) of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$. Then use the second partials test to classify your critical point(s).

10. Let R be the region in the first quadrant bounded by graphs of $y = 2x$, $y = x$, and $y = 4$. Write the double integral as an iterated integral and evaluate.

$$\iint_R (x^2 + y)\, dA$$

11. The first octant space region under the cone $z = \sqrt{x^2 + y^2}$ and above the circle $y^2 = 2x - x^2$ has volume given by

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2}\, dy\, dx.$$

Convert this integral to an iterated integral in polar coordinates. Do not evaluate.

12. Evaluate the line integral $\int_C \vec{F}(x, y) \cdot d\vec{r}$, where $\vec{F}(x, y) = -y\hat{\imath} + x\hat{\jmath}$ and C is the path along $y^2 = 3x$ from $(3, 3)$ to $(0, 0)$.